INVESTIGATING OF AN HOURGLASS

Fatemeh Janmohammadi, Farzanegan 7 High School,Tehran/Iran , f.jani80@gmail.com

ABSTRACT

In this research parameters that are effective on an hourglass has been surveyed. Whether or not the weight of a running hourglass differs from the weight of the hourglass at rest is explained by several experiments.

ARTICLE INFO

Winner of Silver Medal in ICYS 2018, Belgrade, Serbia

Supervisor: Dr. Hossein Salari

Accepted in country selection by Ariaian Young

Innovative Minds Institute , AYIMI

1 Introduction

In order to solve this question, suppose we have a running hourglass which is in its steady state (Fig.1). In this situation on one hand, since a portion of the sand is in free fall, it seems that the hourglass should weigh less. On the other hand, the impact of the sand on the base of the hourglass should increase the downward force exerted on the scale. We want to understand that which effect is greater and how they are acting on each other and investigating some parameters that would have effect on changing the weight of a running hourglass.

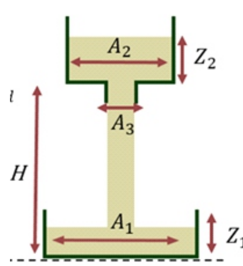


Fig.1: an hourglass in a steady state

2 Theory and Model

We decompose the running hourglass into three states (Fig. 2):

1)the beginning state ,when the hourglass starts to flow until the first bit of sand hits the ground

2)the steady state when both upper and lower chambers contain a portion of sand

3)the end state ,when the last bit of sand leaves the upper chamber until it hits the ground

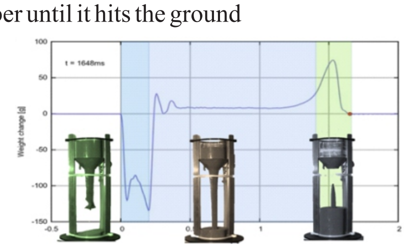


Fig.2: The running hourglass in three states

In the beginning state we have just free fall and we expect the apparent weight of hourglass decreases. In order to find the total time of this state t_1 and the maximum decreasing weight δW_1 we have to take into account some assumptions. First we assume the flow rate Q reach to a constant M/T where M is the total mass of sand in the hourglass and T is the total time of flowing hourglass[2] .

Second, we assume the sand grains do not have any interaction to each other during the falling. With these two assumptions we can easily find t_1 as the following :

$$H = \frac{1}{2}gt^2 \qquad t_1 = \sqrt{\frac{2H}{g}} \tag{1}$$

where (H) is the distance between the orifice and the ground. Now we can find δW_1 as (eq. 2):

$$\delta W_1 = -\delta M_1 g = -Qt_1\, g = -Q\sqrt{2gh} \tag{2}$$

Therefore we predict a linear decreasing of weight from 0 to $-Q\sqrt{2gH}$ during the time of $\sqrt{\frac{2H}{g}}$ in the beginning state.

In the steady state we have both free fall and impulse at the same time. The total time of this state would be t_2 . In this state we assume the impulse is a pure inelastic impact and the momentum of the sand reaches to zero after hit the ground. Imagine in a typical time t the height of z_1 in the lower chamber is filled. Therefore, the total losing weight would be $-Q\sqrt{2g(H-Z_1)}$ and the force of impact can be obtained by:

$$f_{imp} = \frac{\delta P}{\delta t} = \frac{0-(-\delta mv)}{\delta t} = \frac{Q\delta tv}{\delta t} = Qv \ (3) \tag{3}$$

on the other hand, from conservation of energy we have:

$$\frac{1}{2}mv^2 = mg(H-z_1) \tag{4}$$

$$v = \sqrt{2g(H-Z_1)} \tag{5}$$

which by substituting the total force of impact would be $Q\sqrt{2g(H-Z_1)}$ so the net deviation of weight in the steady state would be $\delta W_2 = Q\sqrt{2g(H-z_1)} - Q\sqrt{2g(H-z_1)} = 0$ and in the steady state we expect not changing in weight of hourglass.

In the end state, the amount of sand in the free fall will be decrease by passing time but the impulse will still remain. The total time of this state t_3 can be found by the time of free fall of last grain of sand from orifice to the head of lower chamber (z_1) and would be $t_3 = \sqrt{\frac{2(H-Z_1)}{g}}$. The maximum increasing of weight due to impulse would be:

$$\delta W_3 = Q\sqrt{2g(H - z_1)} \qquad (6)$$

Then, in this state we expect a linear increasing of the weight from 0 to $Q\sqrt{2g(H - z_1)}$ during $\sqrt{\frac{2(H-Z_1)}{g}}$ time.

It is a very simple calculation which shows zero changing weight in the steady state but a very important thing that we missed in calculation is the moving of center of mass. We know that the acceleration of the center of mass of a system is equal to the net force on the system divided by the mass of the system. It can be shown that the acceleration of the center of mass of a system in the beginning and end states is ignorable in comparison with other source of weight changing. Therefore we just focus on the acceleration of the center of mass of a system in the steady state. First of all we suppose in a typical time t, when the height of sand in lower and upper chambers are z_1 and z_2. Also, the area of cross sections of lower chamber, upper chamber and orifice are A_1, A_2 and A_3. We assume the sand grains are distributed uniformly in lower and upper chambers, and also in the free fall distributed in a cylinder with area of A_3. With these assumptions the center of mass of system can be find as[1]:

$$z_{cm} = \frac{\rho}{M}[A_1 z_1 \left(\frac{z_1}{2}\right) + A_2 z_2 \left(H + \frac{z_2}{2}\right) + A_3 (H - z_1)\left(\frac{H+z_1}{2}\right)] \qquad (7)$$

$$M = \rho[A_1 z_1 + A_2 z_2 + A_3\ (H - z_1) \qquad (8)$$

now, we can find the decreasing of mass in upper chamber *mu* by the flow rate as the following:

$$\frac{dm_u}{dt} = -Q \rightarrow \frac{d(\rho\, A_2\, z_2)}{dt} = \rho A_2 \frac{dz_2}{dt} = -Q \qquad \frac{dz_2}{dt} = \frac{-Q}{\rho A_2} \qquad (9)$$

By getting a differentiation over time from equation (8) and substituting equation (9), we have :

$$\frac{dM}{dt} = \rho\left[A_1 \frac{dz_1}{dt} + A_2 \frac{dz_2}{dt} + A_3 - \frac{dz_1}{dt}\right] = 0 \qquad \frac{dz_1}{dt} = \frac{Q}{\rho(A_1 - A_3)} \qquad (10)$$

now by two times derivation from :in equation (7) and substituting equations (9) and (10), the acceleration of center of mass can be found as:

$$\frac{d^2 z_{cm}}{dt^2} = \frac{Q^2}{\rho M}\left[\frac{A_1}{(A_1 - A_3)^2} + \frac{1}{A_2} - \frac{A_3}{(A_1 - A_3)^2}\right] \qquad (11)$$

Since we have A1,A2 > A3 then the acceleration is always greater than zero which means it is upward. Then the changing weight in the steady state will be:

$$\delta W_2 = M\frac{d^2 z_{cm}}{dt^2} = \frac{Q^2}{\rho}[\frac{A_1}{(A_1 - A_3)^2} + \frac{1}{A_2} - \frac{A_3}{(A_1 - A_3)^2}] \qquad 12)$$

3 Experiments

We made the hourglass with two rods, a glass container, a funnel and the gate for controlling the time when we want the grains to pour (Fig. 1).

We filled the upper container with different materials (salt, sand) and we put it on the scale . We took a slow motion film during the experiment .When the sand starts pouring, the weight decreased then It has some fluctuations and after that It increased (able 1).

Fig. 3: Experimental hourglass

Table 1: Measurements in experimental hourglass

Measurement	
Parameter	Magnitude
A_1	$78.5cm^2$
A_2	$33.16cm^2$
A_3	$2.09cm^2$
h	$52cm$
ρ(salt)	1.4 g/cm^3
ρ(sand)	1.8 g/cm^3
$\rho(iron)$	4.9 g/cm^3

The triple beam scale had a short response time but because of its low measurement accuracy we did not see the magnitude of decreasing and increasing of weight that we expected and we did not δW_2 so we made a lightweight hourglass to be able to do our experiment with digital scale that had a good response time and high measurement accuracy, so we saw the magnitude of weight we expected to see δW_2 (Fig. 4 and table 2).

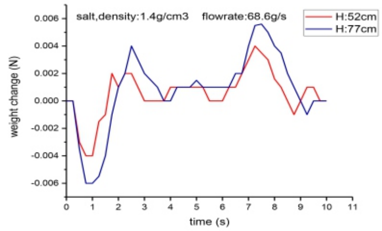
Table 2: Measurements in lightweight hourglass

Measurement	
Parameter	Magnitude
A_1	$23.7cm^2$
A_2	$69.3cm^2$
A_3	$0.78cm^2$
h	$34cm$
ρ(salt)	1.4 g/cm^3
ρ(sand)	1.8 g/cm^3
$\rho(iron)$	4.9 g/cm^3

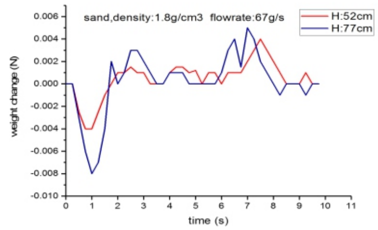
Fig. 4: Lightweight hourglass

4 Results and discussion

We can see that height(distance between orifice and the ground) has an effect on decreasing and increasing of weight. When we have a longer system, the variations of weight would be greater and we can see the same results with different materials . Our materials were sand and salt with different densities which sand is a bit more dense than salt (Fig. 5 a / b; Fig. 6) .



(a)



(b)

Fig. 5: Weight changes versus time in the first hourglass, with different materials a) salt and b) sand

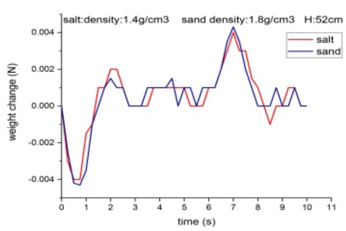
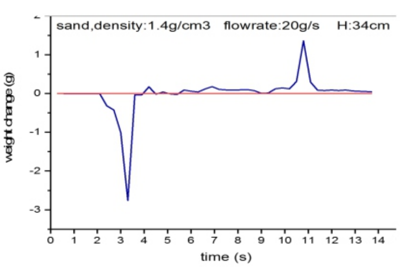
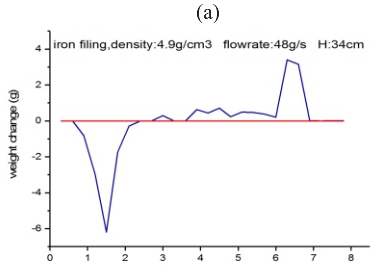


Fig. 6: Weight changes versus time in the first hourglass, comparison of salt and sand

The correct magnitude of changes of weight and δW_2 in two different materials (sand and iron filing) (Fig 7 a/b).



(a)



(b)

Fig. 7: Weight changes versus time in lightweight hourglass with different materials, a) sand , b) iron

5 Conclusion

When we have a material with greater density, fluctuations of weight at first and at the end (even in the steady state that we can not see because of low measurement accuracy of scale) will be greater. In the first setup ,the changes of weight are not exactly match with the numbers in our theory and we did not see δW_2 because of the low measurement accuracy of triple beam scale.

The results from the second setup (the lightweight setup and digital scale) show the correct magnitude of changes of weight and δW_2 in two different materials (sand and iron).

We have some other effective parameters in this phenomenon which can affect on changing the weight of it .By increasing the area of A_1 and A_2, the changing weight will decrease. On the other hand by increasing the area of A_3 the flow rate (Q) will be increased and then the changing weight will be increased too. By increasing the density ρ the flow rate Q will be increased and then the changing weight will be increased too.

References

[1] Haliday D., Resnik R. ,Walker J.,1923. Fundamental of physics.John Wiley and sons.

[2] Sack A., and Pöschel T., 2017. "Weight of an hourglass Theory and experiment in quantitative comparison." American Journal of Physics 85.2 : 98-107