SCIENTIFIC RESEARCH ON WATER BOTTLE FLIPPING

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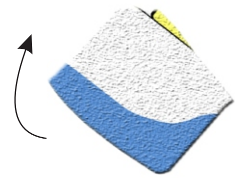

A R T I C L E I N F O

Winner of Bronze medal in ICYS 2018, Belgrade, Serbia

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A B S T R A C T

The current craze of water bottle flipping is a problem of the 31th International Young Physicists' Tournament, involves launching a partially filled plastic bottle into the air so that it performs a somersault before landing on a horizontal surface in a stable, upright position. First the important role of some physical rules were investigated and then according to the theories the existence of some factors which were related to our theories was predicted. Second this flipping with different percentages of water filled was done to compare the best and worst given results together and the purpose of this comparison was to ensure whether our theory in a successful flipping was correct or not and could we explain this phenomenon correctly.

1 Introduction

In 2016, a viral video of a teenager, Michael Senatore, flipping a water bottle at a talent show at Ardrey Kell High School in Charlotte, North Carolina popularized the activity. Senatore had started flipping water bottles the year prior in his chemistry class, and mastered the trick. After his performance, the recorded video became a viral success; the trend spread across the rest of the world, and is still being done as of 2018 [1].

In the first sight we saw the rotation of the bottle similar to the rotation of the badminton ball in the air or diving of a person (Fig. 1).

Fig. 1: The rotation of the bottle

But the remarkable thing we missed was that all of those examples are rigid body while the water inside the bottle turns our object into nonrigid body and definitely the movement of the water causes a lot of changes, including the changes of center of mass or the sloshing of the water will redistribute the mass along the bottle and the changes of the moment of inertia. As a result, that water sloshing has important role in a successful landing, but we don't know how it can affect on the bottle's velocity? How we can explain and investigate the water sloshing while it is complex phenomenon? What are some other effective parameters? What is the optimal filling fraction and why?

2 Theory & Modeling

2-1 Sloshing

The sloshing of the water is one of the effective parameters in lowering the bottle's velocity itself and on the other hand it leads to a redistribution of the mass along the bottle that will increase the moment of inertia which is the second parameter [2]. Sloshing will take the kinetic energy in so far as the bottle will lose its velocity [Eq. 1].

$$K_\omega = \frac{1}{2} I\omega^2 \quad (1)$$

where I is the moment of inertia and ω is angular velocity.

Since sloshing is complicated subject in physic, the changing of center of mass was spotted as a sign which shows the existence of sloshing in the phenomenon.

2-2 The moment of inertia

We know the fact that bottle's angular momentum must be conserved. For a rigid body, the conversation of angular momentum implies a rotation with constant angular velocity making a smooth landing rather unlikely, but in our nonrigid body the redistribution of the mass along the bottle will increase the moment of inertia and then the angular momentum implies a decrease of rotational velocity leaving the impression of the bottle being suspended horizontally in the air for a moment. Then the bottle will fall slowly and land upright.

To explain it from the other side which is easier, the angular momentum can be assumed constant because gravity force is applied to the body and by this assumption the moment of inertia will decrease the angular velocity.

$$L = I\omega \quad (2)$$

2-3 Center of mass

The center of mass in a system is the point that moves as though all of the system's mass were concentrated there and all external forces were applied there [3].

$$x_{cm} = \frac{\sum_{i=0}^{n} m_i x_i}{M} \tag{3}$$

$$x_{cm} = \frac{1}{M}\int x\, dm \tag{4}$$

2-4 Angular momentum

Angular momentum is the quantity of a rotating body which is a product of angular velocity and the moment of inertia. Since there is only gravity force which is applied to our body we assumed that there is no external force and according to Newton's second law in angular form which is "If no net external torque acts on the system, this equation becomes :

$$d\vec{L}/dt = 0$$

so angular momentum would be constant.

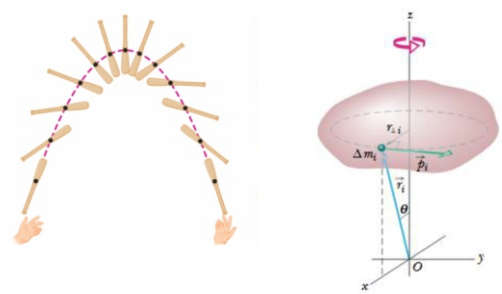


Fig. 2: Angular momentum

2-5 The moment of inertia

The moment of inertia tells us how the mass of the rotating body is distributed about its axis of rotation [3].

$$I = Mr^2 \tag{5}$$

Where, M is the total mass of the body, and r is a radius from the central axis to a particle.

This is used for calculating the moment of inertia for the Object which is about central axis, but since our body is not rotating around its center of mass point we considered that the bottle is about axis which is parallel to the axis through the center of mass and we used parallel axis theorem to calculate the body's moment of inertia.

$$I = I_{com} + Md^2 \tag{6}$$

Where, I_{com} is a moment of inertia of a body which is about an axis through the center of mass point , M is total mass of our body, and d is the distance between axis through the center of mass and axis which is parallel to central axis.

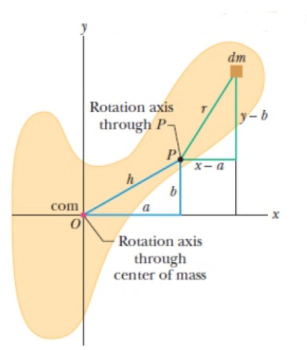


Fig. 3: A rigid body in cross section, with its center of mass at O.

The parallel-axis theorem (Eq.6) relates the rotational inertia of the body about an axis through O to that about a parallel axis through a point such as P, distance h from the body's center of mass. Both axes are perpendicular to the plane of the figure [3].

3 Experiments

Our whole system is separated into two parts :

1. Empty bottle that its center of mass is located at $H/2$

2. Water that its center of mass is located at $h/2$ (when the bottle is self centered and all of the water is at the bottom of the bottle).

By this assumption we approximately could understand the total body's center of mass's position which is shown as h_{cm} in Fig. (4). Notable point is that the water will distribute along the bottle and as a result the height of the water varies from its minimum h_0 to maximum value located H (when water is distributed through the bottle completely) as shown in Fig. 4 The center of mass can be found by Eq.(7) .

$$h_{cm} = \frac{\frac{H}{2}m_b + \frac{h}{2}m_w}{m_b + m_w} = \frac{H}{2}\left(\frac{m_b + m_w\frac{h}{H}}{m_b + m_w}\right) \tag{7}$$

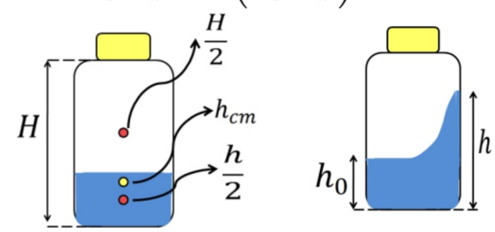


Fig. 4: h is the height of the water which varies with time

According to water's height changes the total center of mass varies and the maximum sloshing can happen when the center of mass reaches the $H/2$ from h_{0cm}.

$$h_{cm}^0 = H/2\left(\frac{m_b + m_w h_0/H}{m_b + m_w}\right) \tag{8}$$

By considering the bottle as a cylinder, the total moment of inertia yields us Eq. (9-11).

$$I_b = I_0 + m_b\left(\frac{H}{2} - h_{cm}\right)^2 \tag{9}$$

$$I_w = \frac{1}{12}m_w h^2 + m_w\left(\frac{h}{2} - h_{cm}\right)^2 \tag{10}$$

$$I = \frac{1}{12}(m_w h^2 + m_b H^2) + m_w\left(\frac{h}{2} - h_{cm}\right)^2 + m_b\left(\frac{H}{2} - h_{cm}\right)^2 \tag{11}$$

In last step by considering the angular momentum constant we come to this conclusion that angular frequency can be expressed as

$$\frac{\omega(t)}{\omega_0} = \frac{I_0}{I(h)} \tag{12}$$

where $I(h)$ is given by Eq.(11) and is the initial angular velocity and I_0 is initial moment of inertia when we have the minimum height of water h_0.

According to the distribution of the mass along the bottle we knew that the moment of inertia will increase and certainly the angular velocity will decrease and this changing in angular velocity can be inferred as a sign of losing velocity in bottle. For further explanation, when ω is equal to zero, means that two consecutive angle are equal to each other or on the other hand the object has lost its velocity.

By using all these facts ω_0 should reduce ω as much as possible so we should look for the minimum of the ratio $I_0/I(h)$ which can obtained when maximum $I(h)$ is attained, and for each filling fraction $f = h_0/H$ the maximum moment of inertia is attained when water is maximally distributed along bottle and $h = H$ [2] . Therefore, we should look for the value of f for I_0/I_{max} .

For finding the I_0/I_{max} we can use more analytical form which first needs to determine the mass ratio.

$$M = \frac{m_{w,max}}{m_b} \tag{13}$$

Where $m_{w,max}$ is the water mass for a filled bottle. With this, the mass of the water can be defined as $m_w = f m_{w,max} = f M m_b$

$$\frac{I_0}{I_{max}} = \frac{M^2 f^4 + 4Mf^3 - 6Mf^2 + 4Mf + 1}{(1+Mf)^2} \tag{14}$$

According to the effective role water sloshing has in the phenomenon, the minimum value of h_{cm}/H can be the sign of maximum changing of the center of mass or on the other hand maximum water sloshing. Again we can find better and more analytical form for finding instead of expression (7) by introducing the mass ratio.

$$\frac{h_{cm}}{H} = \frac{1}{2}\left(\frac{1+Mf^2}{1+Mf}\right) \tag{15}$$

At last we come to conclusion that optimal filling fraction should include the minimum of I_0/I_{max} and also minimum of h_{cm}/H

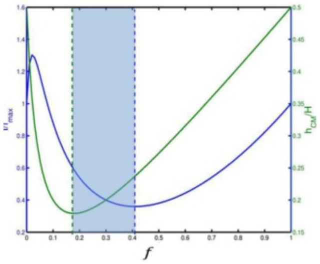


Fig.5: Two green and blue curves are h_{cm}/H and , I_0/I_{max} respectively

The relation between the changing of center of mass and the moment of inertia are shown in Fig. (5) for:

$M = m_{w,max}/m_b = 20.$

As shown in Fig.5 the minimum of h_{cm}/H is in f=0.18 and the minimum of I_0/I_{max} is in f=0.41 , then we can determine the optimal range which is approximately from 20% to 40% of water filled and it is shown as the blue zone in Fig. (5) [2].

4 Results

1- This flipping was done about 120 times with 5%, 20%, 30%, 33%, 40%, 50%, 60%, and 70% of water filled. The best given result was 33% (a) and the worst were 5% (b) and 70% of water filled. Then the angular velocity was calculated by measuring the angles of the bottle in 22 frames and then the changing of angular velocity was investigated to figure out whether it will be equal to zero or not in 33% as the optimal filling fraction.

2- In the first step our optimal range in experiment was compared with theory and the best range is from f=0.18 to f=0.4 which is shown as the blue zone in Fig. (6).

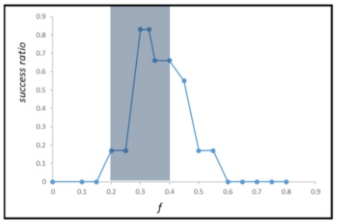


Fig.6: Success ratio in flipping water bottle

3-The next result was about water sloshing and obvious distribution of water along the bottle.

Fig.7: Sloshing in flipping water bottle

4- Decreasing of angular velocity was other result in our experiments which is shown in Fig. (8).

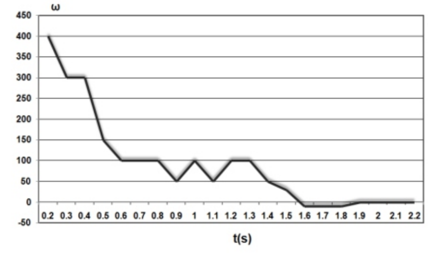
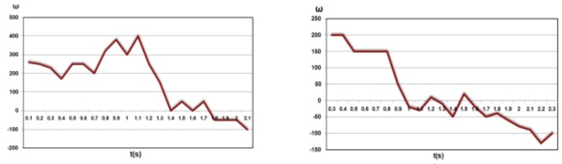


Fig.8: Angular velocity vs. time in flipping water bottle

The angular velocity for 5% (a) and 70% (b) of water filled was measured to find if it decreases or not .



(a) (b)

Fig.9: Angular velocity vs. time in 5% (a) and 70% (b)

5 Conclusion

1-The water sloshing which can defined by changing of the center of mass and the moment of inertia are two effective factors which make the bottle lose its velocity.

First, sloshing will take some rotational energy. second, because of the water sloshing we will have the redistribution of mass along the bottle which increases the moment of inertia, and third since the angular momentum must be conserved during the flipping, the increasing of the moment of inertia will decrease the angular velocity that its decreasing will be expressed as a sign of lowering of velocity which means that bottle will being suspended horizontally in the air for a moment. When performed successfully, the flip ends with a nearly vertical descent that is followed by a smooth landing [2].

2-The optimal range of filling fraction is from 18% to 41% and the optimal single filling fraction is 33% which includes a low value of I_0/I_{max} and h_{cm}/H.

3- The angle of the wrist and hand, the acceleration is given to the bottle, the bottle's shape and volume, the viscosity of the liquid, and etc. are other effective factors in this phenomenon which were considered constant in our experiments.

References

[1] https://en.wikipedia.org/wiki/Bottle_flipping

[2] Dekker P.J., Eek L.A.G. , Flapper M.M. , Horstink H.J.C. , Meulenkamp A.R., van der Meulen J. , Kooij E.S., J.H. Snoeijer,2017.
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[3] Halliday & Resnick, “Fundamentals of physics”