Magnus glider

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Abstract

Stretch the free end of the elastic band and then release the glider. Investigate its motion."
Magnus effect has been seen on many flying balls and cylinders like golf balls and etc. it cause the object to curve away from its principal flight path. In base ball matches when player hits the ball, the ball moves in a certain flight path which is somehow like sagittal diagrams in mathematics. This research concludes an investigation on Bernoulli law and by solving it, a lift for a flying cylinder concerning kutta-Joukowski theorem has been obtained. Experiments were done and by tracking the movement of the glider from the videos that has been taken, diagrams has been drawn and compared to theoretical ones and the result was a flight path for the glider.

Back ground knowledge

At first what does the Bernoulli theorem says? Briefly, Bernoulli theorem is that the fluid pressure decreases at points where the speed of the fluid increases. When something is flying through the air it changes the flow streamlines around it by changing the streamlines some forces are applied to the object. Every flying object basically has 4 forces applied on it:

Weight: is the force of gravity. It acts in a downward direction—toward the center of the Earth.

Lift: is the force that acts at a right angle to the direction of motion through the air. Lift is created by differences in air pressure.

Thrust: is the force that propels a flying machine in the direction of motion. Engines produce thrust.

Drag: is the force that acts opposite to the direction of motion. Drag is caused by friction and differences in air pressure (Fig. 1).

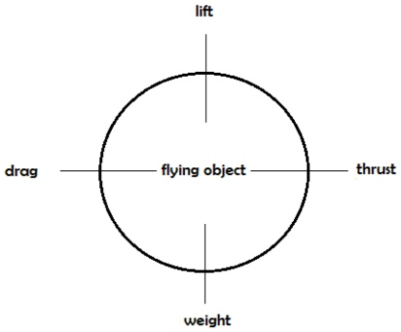


FIG.1: forces on a flying object.

The Magnus glider which has been used in this research is made of two light cups (Fig. 2).

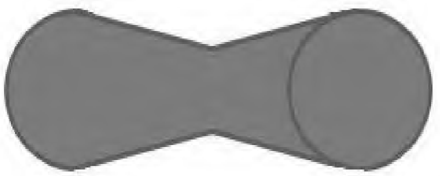


FIG.2 : Magnus glider

Then this glider was considered as two cylinders which the average of both was considered as a cylinder that contains Magnus glider so the first assumption is that the glider is a cylinder with certain accuracy.

Theoretical solution

In FIG.3, a torque will be generated by a resultant vector of force that makes an angle with the axis of rotation of the glider. If the glider has had a sphere ball shape, this torque would cause a motion in a circular path but because of its shape, weight and the axis of rotation which is x , it will deviate from this circular path. But still moves in a semicircular path according to Magnus effect which the lift force is also one of the reasons for gliders motion.

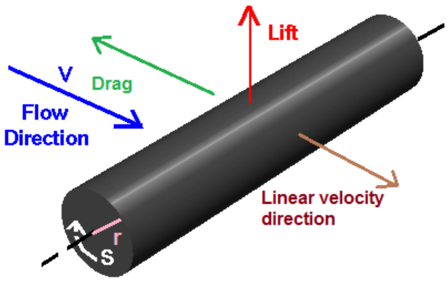


FIG.3: forces and flow directions on glider

First assumptions:

1. Glider is considered as a cylinder
2. In the beginning the flow is considered as incompressible

Assume a cylinder is flying through the air, by its motion it will change the air streamlines around it. It is clear that Reynolds number is important in this change in air flow. Reynolds number explains if the airflow is laminar or turbulent as (Eq. 1) :

$$Re = \rho UD/\mu \qquad (1)$$

By checking the Reynolds number and changes in flow it has been found that the flow around the assumed cylinder is laminar and the flow behind it is turbulent. In a cylinder flying and rotating through the air it will obviously change the air stream lines around it. If the linear velocity is defined as v ,velocity in point a is v+ωr because cylinder is rotating clockwise so in point A rotational speed which is ωr has the same direction with flow stream line. But in point B because rotational speed vector and flow stream lines are in opposite directions so the outcome of these vectors is the result of their subtraction (Fig. 4).

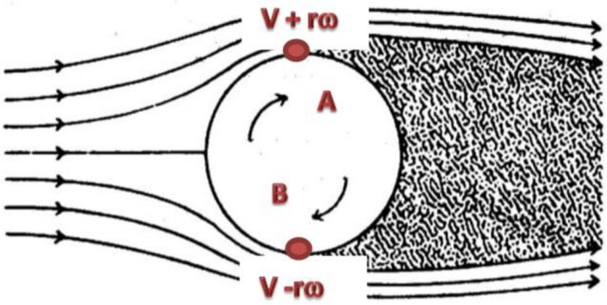


FIG.4: velocity of two points on a vertical line in air

According to Bernoulli, pressure will decrease in points where velocity increases. Velocity in point A is v+ωr and in point B velocity is defined (v- ωr). So velocity in point A is larger than point B therefore pressure in point A is smaller than point B. Flow moves from high pressure to lower pressure. Now due to this pressure gradient mentioned there's an upward force created called lift. The glider is rotating clockwise (Eq. 2).

$$\frac{1}{2}P(v - r\omega)^2 + P_1 = \frac{1}{2}P(v + r\omega)^2 + P_2 \qquad (2)$$
$$\Delta P = \frac{1}{2}P((v + r\omega)^2 - (v - r\omega)^2)$$
$$dF = \Delta P l dU$$

Bernoulli solution

To obtain the pressure distribution from Bernoulli equation after velocity distribution:

$$\frac{P_\infty}{\gamma} + \frac{U^2}{2g} = \frac{P}{\gamma} + \frac{\left(-2U\sin\theta + \frac{K}{2\pi a}\right)}{2g} \qquad (3)$$

$$\frac{P}{\rho} = \left(\frac{U^2}{2}\right) + \left(1 - 4\sin^2\theta - \frac{K^2}{4\pi^2a^2U^2} + \frac{2K\sin\theta}{\pi aU}\right) \qquad (4)$$

$$F_{Lift} = Y = -\int_0^{2\pi} Pa\sin\theta d\theta \qquad (5)$$

$$F_{Lift} = Y = -\frac{1}{2}\rho U^2 a\int_0^{2\pi}\left(\sin\theta - 4\sin^3\theta - \frac{K^2\sin\theta}{4\pi^2a^2U^2} + \frac{2K\sin^2\theta}{\pi aU}\right)d\theta \qquad (6)$$

$$\int_0^{2\pi}\sin^3\theta\, d\theta = \left[-\cos\theta + \frac{1}{3}\cos^3\theta\right]_0^{2\pi} = 0 \qquad (7)$$

$$F_{Lift} = Y = -\frac{\rho UK}{\pi}\int_0^{2\pi}\sin^2\theta d\theta = -\frac{\rho UK}{\pi}\left[\frac{\theta}{2} - \frac{\sin(2\theta)}{4}\right]_0^{2\pi} = -\rho UK = \rho U\Gamma \qquad (8)$$

Thus for the unit length of cylinder lift force would be FL = Y = ρUΓ which is known as Magnus effect and does not depend on the size of cylinder and can be shown that this is not a function of cylinder shape. Meaning that any object that circulation around it is Γ,lift force above is applied.

Of course in actual flow of fluid the amount of circulation generated is a function of shape, size and body condition. Drag force is along the air flow and enters from the flow to object. Pressure drag force for a cylinder with circulation in ideal or Non-viscous fluid is calculated by equation (9).

$$F_{Drag} = X = -\int_0^{2\pi} pa\cos\theta d\theta = 0 \qquad (9)$$

But in actual fluid flow it is not used and is generated by the effects of viscosity. This difference between actual flow and ideal flow is called D'Alembert's paradox in actual fluid with rotating cylinder a circular flow around the cylinder can be generated. As a result the flow around the cylinder is with vortex and the lift force is created. This issue is even seen in other objects motion such as a ball rotating and is used to navigate ships by wind. In this research the equation used for calculating the drag force is given by equation (10).

$$F_D = \frac{1}{2}\rho v^2 C_D A \qquad (10)$$

Where

F_D is the drag force,

ρ is the density of the fluid

v is the speed of the object relative to the fluid,

A is the cross sectional area, and

C_D is the drag coefficient – a dimensionless number

Kutta-Joukowski theorem for a cylinder

$$L = \rho G V \qquad (11)$$

where

ρ is gas density

G is vortex strength, $G = 2\pi b v_r(*)$

V is velocity(m/s)

$(*)v_r$ is the rotational speed ,$v_r = 2\pi b s$

s is spin (rad/ second)

b is radius of cylinder

As you see Bernoulli and Kutta-Joukowski reach the same lift formula.

Equations of Motion

The equation of motion is written based on the formulas mentioned before. Cylinder has a projectile motion and starts it motion with an initial velocity makes an angle with the horizon line. Like the figure (5) velocity and every action and reaction forces has been decomposed into its horizontal and vertical components. Then by writing equation of forces in each axis and solving , motion equations have been obtained and lift and drag forces are calculated from Kutta-Joukowski theorem and drag in actual flow.

Theoretical diagrams

In this stage motion equation are used to get a predicted diagram of the glider motion which is simply close to a motion (Fig. 6).

$$\sum Fx = -L_y - D_x$$

$$\sum Fy = L_x - (D_y + W)$$

$$F_{net} = ma \quad \Rightarrow \quad a = \frac{F_{net}}{m}$$

$$a_x = \frac{-L_y - D_x}{m} \qquad a_y = \frac{L_x - (D_y + W)}{m}$$

$$v_x = a_x t + v_{x\,n-1} \qquad v_y = a_y t + v_{y\,n-1}$$

$$x = v_x t + x_{0\,n-1} \qquad y = v_y t + x_{0\,n-1}$$

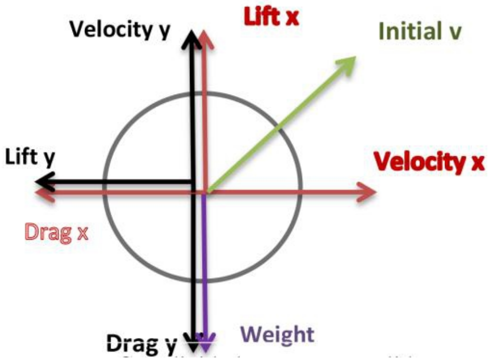


Fig. 5: action and reaction forces

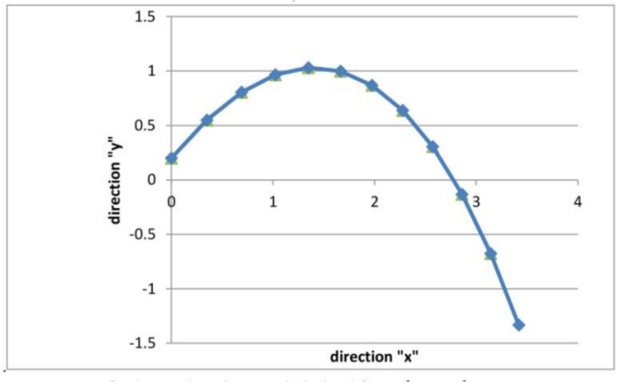


Fig. 6: graph derived from theory data

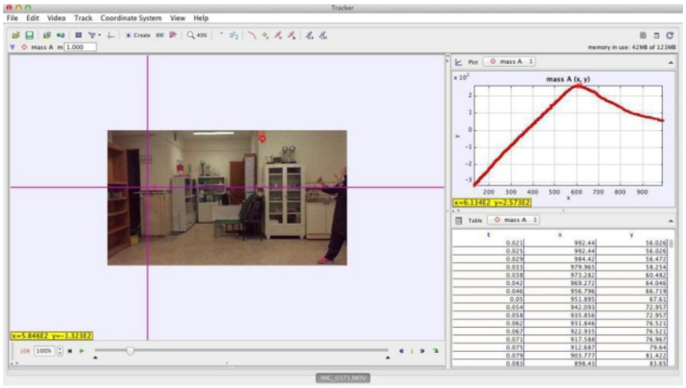
In this predicted graph the origin of coordinates was considered 1.7 m above the ground with the initial velocity of 5 meters per second. The object will start moving with a minus acceleration(retarding motion) and in peak point it reach velocity approximately equal to zero and then continue with a positive acceleration(quicken motion) till it reach the ground and stop moving, so it won't reach zero velocity (stop moving) before collision with ground .

Experiments

Using 2 light cups and a power adhesive tape, glider was made and rubber bands that give the initial force for navigating the glider were made by knotting them to each other. Like the figure (7) rubber bands are twisted around the glider and by releasing the free end glider starts its movement.

FIG.7: process of making a simple Magnus glider.

By tracking a video of experiments with "tracker" program, the location was drawn (Fig. 8).



t	x	y
0.021	992.44	56.026
0.025	992.44	56.026
0.029	984.42	56.472
0.033	979.965	58.254
0.038	973.282	60.482
0.042	969.272	64.046
0.046	956.796	66.719
0.05	951.895	67.61
0.054	942.093	72.957
0.058	935.856	72.957
0.062	931.846	76.521
0.067	922.935	76.521
0.071	917.588	76.967
0.075	912.687	79.64
0.079	903.777	81.422
0.083	898.43	83.65

Fig. 8: screenshot of "tracker" program

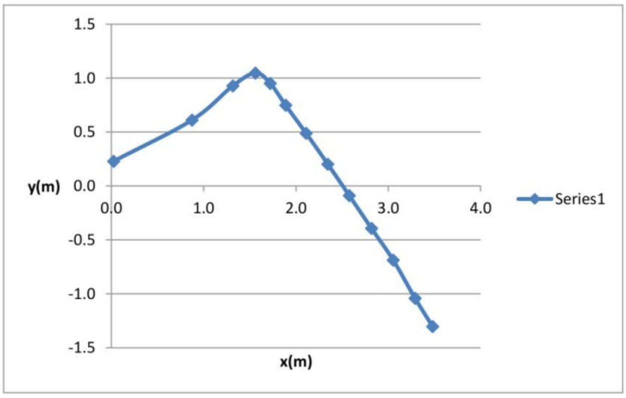


Fig. 9: graph derived from an experiment video (13 points are used from 445 data)

As shown on the figure (9), glider travels an upward path with a decrease in velocity (which is because of the predominance of drag force and other surface frictions) until it reaches the top part where linear velocity approaches zero but still acceleration exist. After that continue its path with an increase in the velocity (in this state weight force overcome the lift force) till it collide with the ground and stop moving.

Relative parameters are:
a) if the glider weight is higher then it will decreases the lift force
b) low initial velocity causes lower acceleration
c) high density of fluid causes to decrease velocity and angular velocity
d) the shape of objects changes the motion

Some of the errors are:
a)Possibility of wind blowing or turbulence in air flow around the glider which causes errors in the gliders motion and deviation from the path
b)Holding the glider and the rubber band around it in a wrong way (too up or too low,..) that cause the wrong motion in glider
c)Existence possibility of inequality and roughness on the glider surface that increase the friction and change the path of motion
And other environmental errors

Comparison between theory and video processing has been shown in figure (10). The general form of two diagrams are the same even the peak points are very close to each other and simply the same but in some points such as the distance between starting point and the peak ,theoretical diagram act a bit more smooth and parabola shape but in experimental diagram data are more factual .

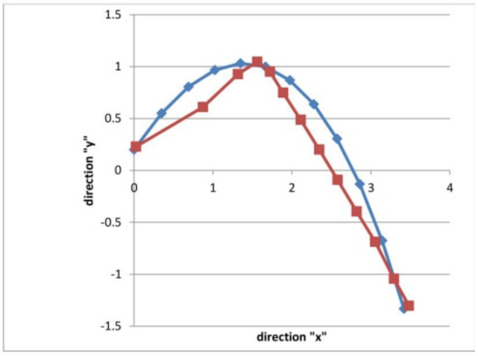


Fig. 10: comparison between theoretical and experimental data

Conclusion

- Magnus effect changes the motion behavior of a thrown object in a projectile motion and creates more lift like forces that make the glider to fly higher.
- The glider will fly with a decreasing velocity caused by drag force and frictions and reaches a point where the glider has the velocity approximately equal to zero and

then continue the path with approximately constant velocity and won't stop before colliding with the ground.

- Lower weight cups can make the glider to fly much higher than heavier ones.
- By considering the flow around the glider semi laminar, a path can be drawn for the glider movement.
- Three forces are applied on the glider: drag, lift and weight, and also because the glider starts moving with velocity and not a force then glider has no thrust.

References

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