Ball in a Tube

* Mobin Moradi
School: Allameh Helli 1

* Corresponding Author: safa_mr99@yahoo.com

Abstract

In this article we want to analyze the motion of a ball in a rotating tube of water .We start with calculating the pressure in a rotating tube and finding the overall shape of water in rotating cylinder ,considering the conservation of the water inside it. Finally, after finding the volume that is occupied by the ball in the water ,We write the motion equations for the ball .Here ,we figure out that when the tube is accelerating the ball starts oscillating on a straight line related to the tube and this oscillating motion is extremely dependent on the viscosity number of the fluid or generally speaking, it depends on the coefficient of the friction force .The other thing that happens is that when we put the ball inside the rotating tube (with constant speed)the ball oscillates around it's equilibrium point and its motion damps and gets steady in its equilibrium point eventually .In our experiment we accelerate the cylinder and find out the oscillation of the ball and we also measure the equilibrium point of the ball and compare it to our theoretical data and plot the graph related to the shape of water and the equilibrium point of the ball in respect to our known parameters .Generally, we can completely describe the motion of the ball by solving the motion equations numerically .During this process we use our knowledge of geometry and calculus to find out the relation between the parameters.

Keywords

Ball, Oscillation, Rotating tube, fluid dynamics

Introduction

In this research we studied the motion of a ball in an inclined tube, which is filled with a liquid and moves in a conical surface.
The general form of fluid's movement is shown by Navier-Stokes equation. There are forces acting on the ball such as Gravity, centripetal force, and buoyancy (the force acted from the fluid) which should be investigated by analyzing the experiments.

Experiment and Theory

According to the Navier- Stokes equation, general form of fluid's movement is found .

$$\rho\left(\frac{\partial}{\partial t}+\boldsymbol{v}.\boldsymbol{\nabla}\right)\boldsymbol{v}=\rho\boldsymbol{g}+\boldsymbol{\nabla}p+\frac{\mu}{3}\boldsymbol{\nabla}(\boldsymbol{\nabla}.\boldsymbol{v})+\mu\nabla^2\boldsymbol{v}$$

Where 'ρ' is density of fluid, '$\boldsymbol{v}$' the velocity of our fluid, 'μ' viscosity coefficient and '$\boldsymbol{g}$' the gravitational field in that area. Since the velocity is constant, the differential velocity is zero so we can write:

$$\rho\boldsymbol{g}+\boldsymbol{\nabla}p=0:\ \vec{g}=-g_0\,\hat{z}+r\omega^2\hat{r}\ ,$$

$$p=p_0+\frac{1}{2}[\omega(r-r_0)]^2\rho-\rho g(z-z_0)$$

The pressure in the rotating cylindrical tube in respect to its radiuscan be found.
Now we are going to find the shape of water when the tube is rotating; so we write the motion equations for a differential matter on the surface of the water.

$$dF\,cos\theta=gdM\ ,\ dF\,sin\theta=dM\,r\omega^2$$

By dividing the two equations:

$$tan\theta=\frac{dz}{dr}=\frac{r\omega^2}{g}\quad\Rightarrow\quad z-z_0=\frac{r^2\omega^2}{2g}$$

This equation gives us the general form of the surface function of the water. All we need to find is z_0 .

We define a new parameter $l = L + z\,cot\varphi$ (Fig.1) and from the geometry of the shape we can write:

$$r_2 = L + \frac{r_2^2\omega^2}{2g}cot\varphi$$

$$r_2 = \frac{g}{\omega^2 cot\varphi}\left(1 \pm \sqrt{1 - \frac{2L\,cot\varphi\,\omega^2}{g}}\right)$$

$$z_1 = \frac{{r_1}^2\omega^2}{2g} + z_0 \Rightarrow z_0 = r_1 tan\varphi - \frac{{r_1}^2\omega^2}{2g}$$

According to conservation of volume:

$$V = \int_{r_1}^{r_2+r_1} \frac{1}{4}\pi(l-r)^2 dz + \frac{z_1\pi L^2}{4} =$$

$$\int_{r_1}^{r_2+r_1} \frac{1}{4}\pi(L + Zcot\varphi - r)^2\frac{r\omega^2}{g}dr + \frac{r_1 tan\varphi\,\pi L^2}{4}$$

$$\Rightarrow \boxed{V_0 = \int_{r_1}^{r_2+r_1} \frac{1}{4}\pi\left(L + \left(\frac{r^2\omega^2}{2g} + r_1 tan\varphi - \frac{{r_1}^2\omega^2}{2g}\right)cot\varphi - r\right)^2\frac{r\omega^2}{g}dr + \frac{r_1 tan\varphi\,\pi L^2}{4}}$$

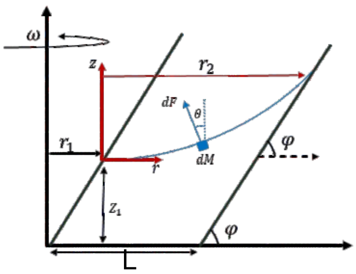


Fig.1-shape of water in the cylinder

By solving this equation z_0 numerically could be found.

Here, we should calculate the water floating in the tube .First ,we use Archimedes law to find the volume that is occupied by the ball (fig.2).

$$Mg = \rho_w v g \Rightarrow v = \frac{M}{\rho_w}$$

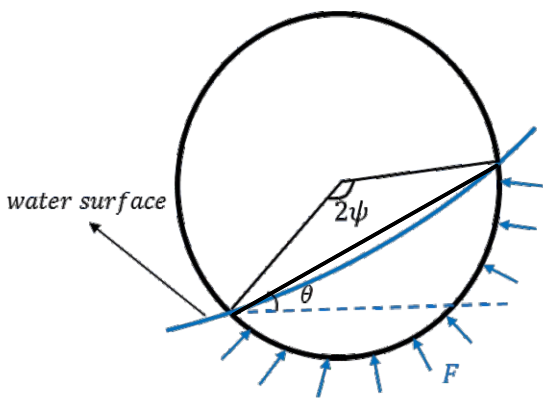


Fig.2-The amount of water occupied by theball

The area of a skullcap could easily calculate.

$$v_{sc} = \int_0^{2\pi}\int_0^{\psi} R^2 sin\theta\, d\theta\, d\varphi - \frac{1}{3}\pi(Rsin\psi)^2(rcos\psi)$$

$$v_{sc} \cong v \Rightarrow 2cos\psi + \frac{sin\psi^2\,cos\psi}{3} + \frac{M}{\rho_w\pi R^3} - 2 = 0$$

$$\Rightarrow v_{sc} = \pi R^3\left(2 - 2cos\psi - \frac{sin\psi^2 cos\psi}{3}\right)$$

As you see we use an approximation that the occupied volume is nearly equals to the volume of the skullcap. Figure (3) describes figure (2) more.

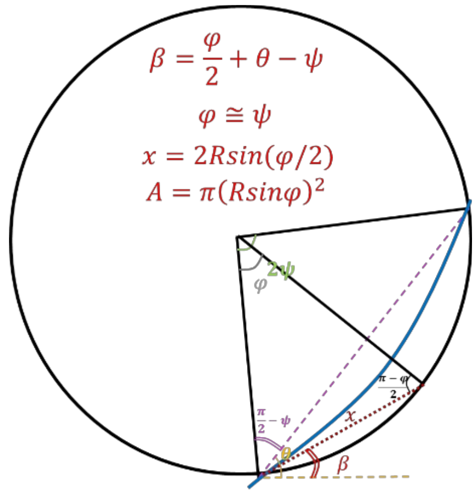


Fig.3-A geometrical sketch of finding the net force

The net force in the fig.3 is on the bisector of : 2ψ

$$tan\theta \cong \frac{dz}{dr} \Rightarrow sin\theta = \frac{1}{\sqrt{1 + (\frac{dr}{dz})^2}}$$

$$F_x = \left[\frac{1}{2}\rho\omega^2\{x\,cos\beta + r\}^2\right]\pi sin\theta(Rsin\psi)^2$$

and also We will take $-M\alpha V\,\hat{V}$ as the friction force applied to the ball and write the movement equations of the ball by using $\hat{V} = \vec{V}/|V|$ .so we can analyze movements of the ball completely.

$$\boxed{\ddot{r} - r\omega^2 = -\frac{1}{M}\alpha\dot{r} - \left(\frac{1}{2M}\rho\omega^2\pi sin\theta(Rsin\psi)^2\left\{r + Rsin\psi\cos\left(\theta - \frac{\psi}{2}\right)\right\}^2\right)}$$

If the ball's velocity related to water becomes zero, then it's at equilibrium ,so:

$$2Mr = \frac{\rho\pi(Rsin\psi)^2\eta\omega^2}{\sqrt{(g^2+\eta^2\omega^4)}}\left\{r + \frac{gRsin(\psi)\cos\left(\frac{\psi}{2}\right)}{\sqrt{(g^2+\eta^2\omega^4)}} + \frac{\eta\omega^2 Rsin(\psi)\sin\left(\frac{\psi}{2}\right)}{\sqrt{(g^2+\eta^2\omega^4)}}\right\}^2$$

$$\eta := r - Z_1 cot\varphi$$

$$\alpha = 6\pi\eta R(1 - cos\psi) \quad (stoke's\ formula)$$

Materials and Method

To do our experiment we need some equipment (fig.4).First we make a hole in a carton. To make the carton steady so that it doesn't bend we connect a wooden board above and attach them to the motor by the shaft. We stick two Styrofoam to the board and connect the sticks and the bottle with an elastic band. By changing the place of sticks the angle of the bottle in respect to the horizon changes and we can do our experiment in a range of angles .Now, to control the velocity of the cylinder we use a dimmer and to find ω we calculate the frequency of the cylinder with camera and multiply it to 2π .

By solving the previous differential equation numerically we get to an important conclusion ;that the motion of the ball damps when our cylinder is accelerating ,and this motion is dependent on the coefficient of friction force. We can observe this phenomenon in our experiment several times .We can also find equilibrium radius of the ball, the shape of the water and the value of the Z $_1$ from equation above and compare it to the experimental data(fig.4).

Fig.4-experiment equipment

Experiments and analyzing data are explained and found completely in Figures(5,6, & 7).

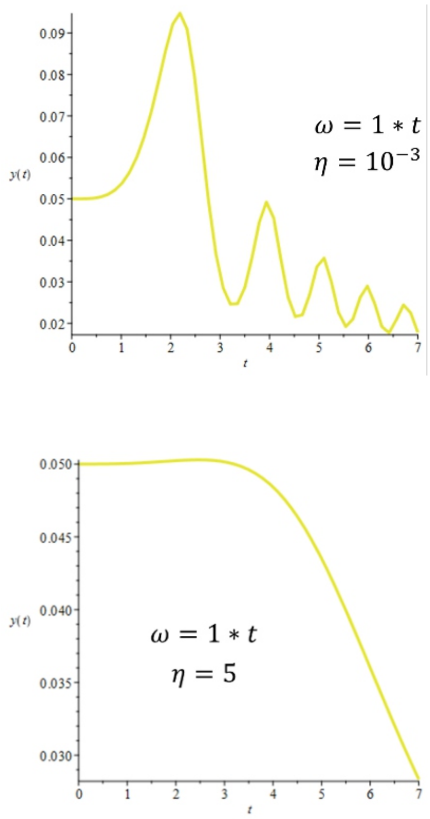


Fig.5-How the motion damps in two different viscosity number

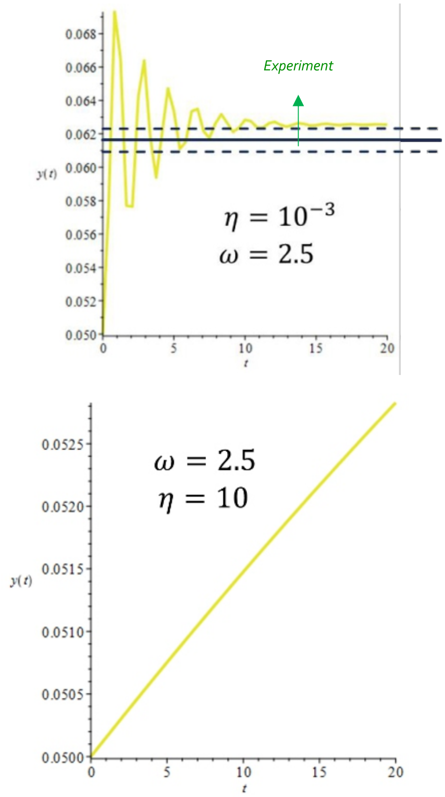
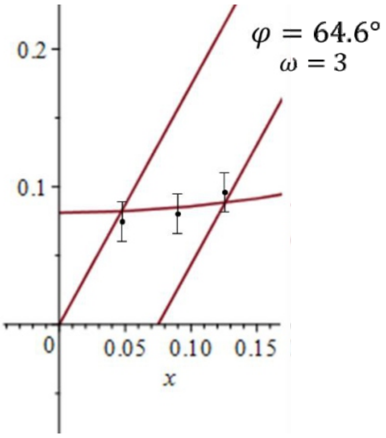


Fig.6-the equilibrium radius of the ball for two different viscosity number (omega is constant)



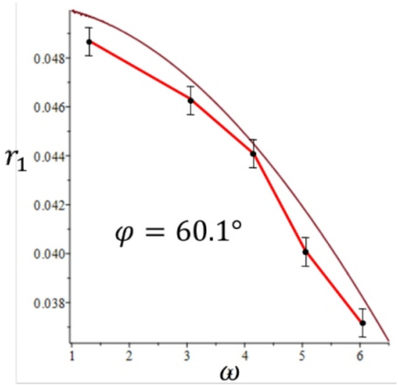


Fig.7: the value of r1 respect to the omega-the shape of the water

Conclusion

After analyzing balls motion theoretically and experimentally we found that the ball has an oscillating movement and that there is a relation between the motion and coefficient of the ball. Meanwhile some of the errors were:

- § We ignored the density changes
- § The integration elements weren't completely circular
- § Approximation of volume (in Archimedes law)
- § Straightness of the tube
- § The surface of the tube isn't completely on the ground
- § Ball changes the shape of water in tube

But despite of these errors you can see that the motion of the ball was well described by related equations.

References

1. Fluid mechanics : Amir Aghamohammadi

2. Fundamentals of Physics, 6/E by Halliday, Resnick, and Walker

IYPT 2017 problem

A sealed transparent tube is filled with a liquid and contains a small ball. The tube is inclined and its lower end is attached to a motor such that the tube traces a conical surface. Investigate the motion of the ball as a function of relevant parameters.