SAXON BOWL

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ABSTRACT

Saxon bowl is a bowl with a hole in its base that will sink when placed in water. In this paper, this phenomenon and the parameters that determine the time of submersion were investigated. Two differential equations were evaluated by neglecting some effects and parameters such as viscosity, water turbulence, etc. Then they were solved either by approximation and coding in Python programming language. While adjusting the vessel mass and hole area, total 54 experiments were conducted by adjusting one variable and keeping the other constant. It was observed that the experiment and modeling results differ but have a close trend because of the neglected parameters and effects.

Key Words: *Saxon Bowl, time measurement, sinking, numerical modeling*

1 Introduction

Saxon Bowl with a hole in its bottom placed in water by Saxons and the time it took the bowl to submerge was measured. The rate in which the bowl sinks depends on several parameters such as the flow rate of water through the hole, pressure, buoyancy, viscosity and etc. To investigate and analyze this phenomenon, experiment and modeling help us to find the submersion time as a function of these factors.

2 Theory and Modeling

The vessel can be modeled as a cylinder with height (L), base area (S) and hole area (A) (Fig. 1). The thickness of the vessel can be neglected. The sunk length of the vessel (h) and the height of water inside the vessel (x) are variables in respect to time.

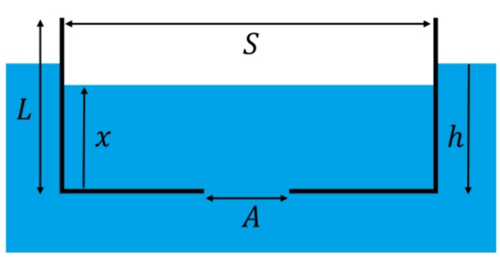

Fig. 1: Modeling of the vessel

To find the flow of water inside the vessel, viscosity of water is neglected and the Bernoulli's equation was used and we assume there's no turbulence in the water flow. Since we neglected the thickness of the vessel, the water viscosity can be neglected in the hole section. The average of parameters and their errors were found by the equations (1-4).

$$\bar{x} = \frac{\sum x_i}{n} \quad (1)$$

$$\Delta\bar{x} = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n(n-1)}} = \frac{\sigma_n}{\sqrt{n-1}} \quad (2)$$

$$\Delta\bar{M}^{-0.5} = \frac{\Delta\bar{M}}{2\sqrt{\bar{M}^3}} \quad (3)$$

$$\Delta\bar{A} = \frac{\pi\bar{D}\Delta\bar{D}}{2} \quad (4)$$

With dimensional analysis, a dimensionless parameter (c) was evaluated which equals to 8.2×10^{-6}. Dimensional analysis shows that the water viscosity in the vessel is relatively small in respect to other parameters of the vessel, so its influence can be neglected (Eqs. 5-7).

$$C = \frac{\mu}{\rho_{water}\sqrt{g\sqrt{S^3}}} = 8.2 \times 10^{-6} \quad (5)$$

$$P + \rho_{fluid} g h + \frac{1}{2}\rho_{fluid} v^2 = Const. \quad (Bernoulli's\ equation) \quad (6)$$

$$Av = const. \quad (Equation\ of\ continuity) \quad (7)$$

In equations, *P* is the water pressure, *h* height in respect to ground, *v* the velocity of water and *A* is the pipe area.

The vessel is sinking inside a bigger vessel. The water flow is modeled as it's flowing from a very wide pipe (bigger vessel) into a smaller pipe (sinking vessel). Since the area of the first pipe is much wider than the second pipe, the equation of continuity concludes that its water flow velocity is relatively small in respect to the smaller pipe so we neglect the third term in the Bernoulli's equation. The third term of the equation for the sinking vessel is found by the water pressure difference at the entrance of the vessel which equals to the static pressure difference between the water inside the vessel with height (x) and the water inside the bigger vessel at depth (h). Moreover, since the sinking vessel is accelerating, the static water pressure inside the vessel decreases and the sinking velocity of the vessel itself is added to the water flow velocity at the entrance (Eqs. 8,9).

$$\frac{1}{2}\rho_{water}(v_{entrance} - \dot{h})^2 = \rho_{water} g h - \rho_{water}(g - \ddot{h})x \quad (8)$$

$$A v_{entrance} = S\dot{x} \quad (9)$$

With some substitutions, the rate of water flow inside the vessel is found (Eq. 10):

$$\frac{dx}{dt} = \frac{A}{S}\left(\sqrt{2(gh - (g - \ddot{h})x)} + \dot{h}\right) \quad (10)$$

buoyancy force, drag, and the change in momentum flux of water flow when it's entering the vessel from the hole (Eq. 11).

$$\vec{F} = M\vec{a} = \frac{d\vec{P}}{dt} \quad (11)$$

Since the density of the Polypropylene (PP) is nearly as water, the buoyancy force of the vessel material itself, cancels its weight. Coins were used to increase the mass of the vessel, which their density is more than water's density but the buoyancy force and the acceleration of the vessel itself, decreases the force they exert on the vessel. The buoyancy force that exerts on the system is found by the water pressure difference at the entrance of the vessel which equals to the static pressure difference between the water inside the vessel with height (x) and the water inside the bigger vessel at depth (h).
The drag force is found by equation (12) where C_D and S-A are the coefficient of drag and the projected area, respectively. The coefficient of drag was found from figure (2) which is the coefficient of drag in respect to the Reynolds number (Eq. 13).

$$F_D = \frac{1}{2} C_D \rho_{fluid} (S-A) v^2 \quad (12)$$

$$R_E = \frac{\rho_{fluid} v D}{\mu} \quad (13)$$

The Reynolds number for the vessel is approximately 700. So from figure (2), the coefficient of drag for the vessel is approximately 1.

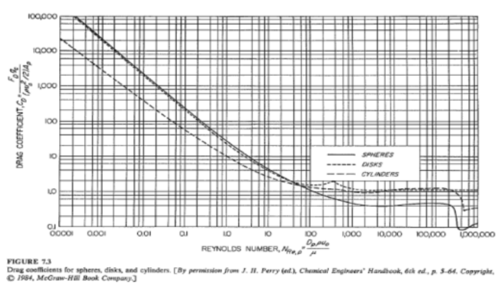

Fig.2: Coefficient of drag chart

Momentum flow for a stream of water is described as follows (Eq. 14):

$$\dot{\vec{P}} = \rho A v^2 \hat{v} \quad (14)$$

For the vessel, the momentum flows are (Eqs. 15-17):

$$\dot{\vec{P}}_{in} = \rho A (v_{entrance} - \dot{h})^2 \hat{z} \quad (15)$$

$$\dot{\vec{P}}_{out} = \rho S (\dot{x} - \dot{h})^2 \hat{z} \quad (16)$$

$$\vec{F} = \dot{\vec{P}}_{in} - \dot{\vec{P}}_{out} = \left(\frac{S}{A}\dot{x}^2 - \dot{h}^2\right)(S - A)\,\hat{z} \quad (17)$$

Then Newton's second law will be (Eq. 18):

$$(M_{coins} + M_{vessel})\ddot{h} = \left(1 - \frac{\rho_{water}}{\rho_{coin}}\right) M_{coins}(g - \ddot{h}) - \rho_{water}(S-A)\left((gh - (g-\ddot{h})x) - \frac{1}{2}\dot{h}^2 + \frac{S\dot{x}^2}{A}\right) \quad (18)$$

and the equations of the vessel are (Eqs. 19, 20)

$$\dot{x} = \frac{A}{S}\left(\sqrt{2(gh - (g - \ddot{h})x)} + \dot{h}\right) \quad (19)$$

$$\ddot{h} = \frac{\left(1 - \frac{\rho_{water}}{\rho_{coin}}\right) M_{coins} g - \rho_{water}(S-A)\left(g(h-x) - \frac{1}{2}\dot{h}^2 + \frac{S\dot{x}^2}{A}\right)}{\left(M_{vessel} + \left(2 - \frac{\rho_{water}}{\rho_{coin}}\right) M_{coins} + \rho_{water}(S-A)x\right)} \quad (20)$$

3 Experiments

A 11.290 (g) Polypropylene (PP) vessel with the dimensions of 12 (D) * 8.5(H) (cm) and with a central hole was used. The weight of vessel was adjusted by adding coins on the base. The mass of the vessel and coins were measured by a balance with accuracy ± 0.001(g), the time of submersion by a stopwatch with accuracy ± 0.01(s) and the vessel's dimensions by a ruler with accuracy ± 0.1(cm) (Fig. 3).

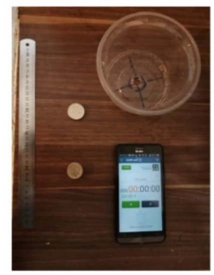

Fig. 3: Experimental setup

In this experiment, two variables of vessel mass and hole diameter were used, ranging from 31.530 to 88.984 (g) and 1.6 to 3.4 (cm), respectively. The method was placing the vessel so that the hole was just in contact with the water surface. The vessel was allowed to sink while the time of submersion was recorded until the surface of water reached the lip of the vessel.
To investigate the influence of various parameters i.e. vessel mass and hole diameter, while the hole diameter was 1.6 (cm), for 9 masses, 27 experiments and while the mass was 82.222(g), for 9 diameters, 27 experiments were conducted.

4 Results and Discussion

4-1 Solution I

The equations 19 and 20 are solved analytically with some approximations and simplifications. Since the vessel is sinking with low velocity, it is assumed that the changes in x and h parameters occur slowly, with the result that the system is always in (approximate) equilibrium. So it's concluded that the vessel is sinking with constant velocity and no acceleration (Eq. 21):

$$\begin{aligned} \ddot{h} &= \ddot{x} = 0 \\ \dot{x} &= \dot{h} = v = Const. \\ h - x &= \ell = Const. \end{aligned} \quad (21)$$

then:

$$v = \frac{A}{S-A}\sqrt{2g\ell} \quad (22)$$

$$\left(1 - \frac{\rho_{water}}{\rho_{coin}}\right) M_{coins} g = \rho_{water}(S-A)\left(g\ell + v^2\left(\frac{S}{A} - \frac{1}{2}\right)\right) \quad (23)$$

$$\ell = \frac{(S-A)\left(1 - \frac{\rho_{water}}{\rho_{coin}}\right) M_{coins}}{\rho_{water} S^2} \quad (24)$$

$$v = \sqrt{\frac{2\left(1 - \frac{\rho_{water}}{\rho_{coin}}\right) g M_{coins} A^2}{\rho_{water}(S-A)S^2}} \quad (25)$$

The height of the vessel is L, and the initial (t=0) value of h was found by buoyancy force of the system when there's no water in the vessel (equation 32). So the time of submersion T is as follows (Eqs. 26-27):

$$h_0 = \frac{M_{coins}}{S\rho_{water}} \tag{26}$$

$$T = \frac{L - h_0}{v} = \left(L - \frac{M_{coins}}{S\rho_{water}}\right)\sqrt{\frac{\rho_{water}(S-A)S^2}{2\left(1 - \frac{\rho_{water}}{\rho_{coin}}\right)g}}\,\frac{1}{A\sqrt{M_{coins}}} \tag{27}$$

Assuming that the hole area is much smaller than the base area of the vessel $(S \gg A)$ and the initial value of h is much smaller than the height of the vessel $(L \gg h_0)$, the time of submersion is related to the hole area by A^{-1} and it's related to the coins mass by $M^{-0.5}$ (Eq. 28).

$$T \propto \frac{1}{A\sqrt{M_{coins}}} \tag{28}$$

The experiment results are plotted (Figs. 4 and 5). To investigate the conclusion of equation (28), the time of submersion was plotted as a function of $M^{-0.5}$ and A^{-1} which fitted the trend line with R-squared value of 0.98 and 0.94, respectively.

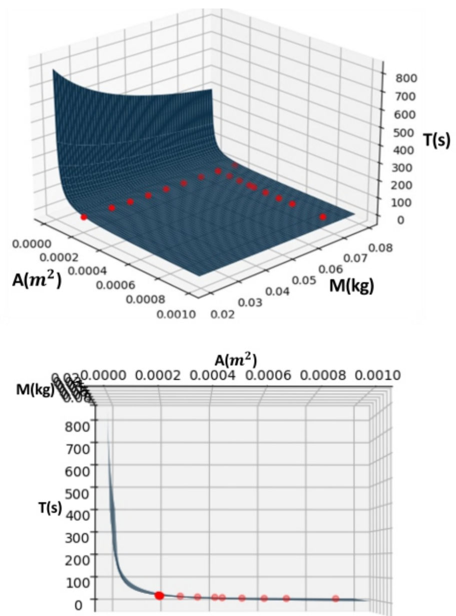

Fig. 4 and 5 : Time of submersion as a function of coins mass and hole area for solution (I) compared to experiments

and figures (6-9):

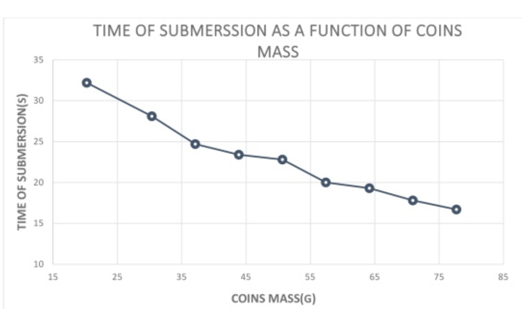

Fig. 6: Time of submersion as a function of coin's mass

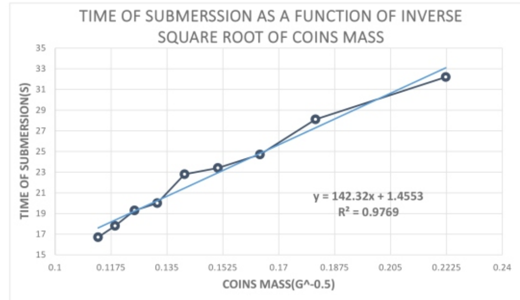

Fig. 7: Time of submersion as a function of inverse square root of coin's mass

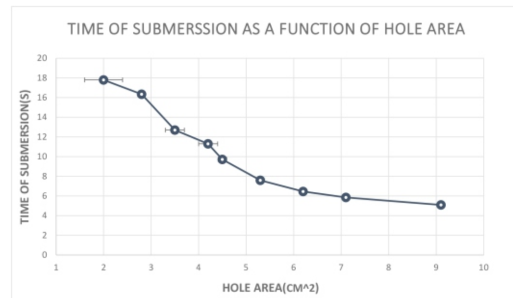

Fig. 8: Time of submersion as a function of hole area

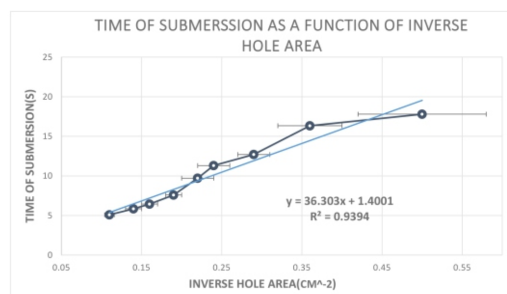

Fig. 9: Time of submersion as a function of inverse hole area

4-2 Solution II

Another way to solve the equations 19 and 20, is to solve them numerically. So Python programming language was used to write a code that solves the equations numerically. So the properties and initial conditions of the vessel should be defined in the code.

The code plots both vessel parameters x and h as a function of time. So the solution of the equations 19 and 20 and the vessel motion tracked by the Tracker program is as follows (Fig. 10):

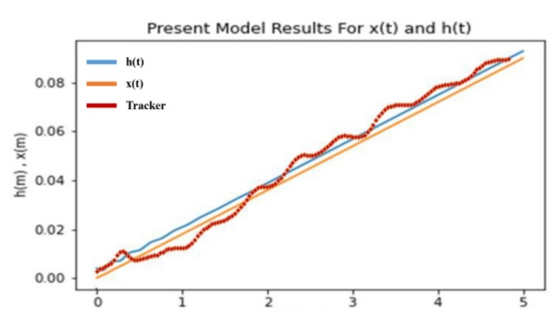

Fig. 10: The numerical solution for x(t) and h(t)

To find the submersion time as a function of hole area and coin's mass, syntaxes were added to the code to find the time where $\boldsymbol{h = 8.5\ (cm)}$ for each coin's mass or hole area and plot them as a function of coin's mass or hole area and then plot the experiment results, so they could be compared (Figs. 11 and 12):

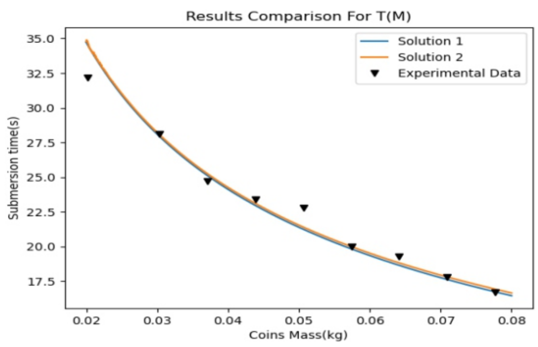

Fig. 11: Results comparison for submersion time as a function of coin's mass

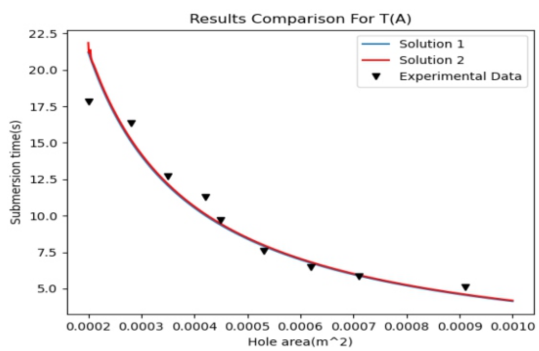

Fig. 12: Results comparison for submersion time as a function of hole area

4 Conclusion

The results show that the trend of the experiment results and the model results are close. In the modeling, some parameters and effects such as viscosity (ignored), coefficient of drag (approximated), water turbulence (ignored), water surface tension (ignored), etc. were neglected. So it's expected that the experiment and modeling results differ but have a close trend as it can be seen in the graphs.

References

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[2] Halliday, Resnick, Walker, Fundamentals of physics