TRAJECTORY OF A ROLLING OBJECT ON A ROTATING DISC

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ABSTRACT

In this paper, we survey on the motion of a rolling object on a turntable exhibiting trajectories without being expelled from the disc like revolving in a stable circular orbit or coming toward the center of the disc that depends on initial conditions and range for observation of a specific trajectory is limited. Based on references frequency of the turntable and rolling friction force are the most important parameters affecting on this phenomenon. According to our experiments both of frequency of the turntable and rolling friction force can effect on velocity and mass center velocity of the rolling object that in some of situations make the rolling object come toward the center.

Keywords : *rolling object, rolling friction, frequency of turntable*

1. Introduction

When a rolling object (sphere, disc, ring,…) is put on a horizontal rotating disc although we predict that the rolling object will be cast away from the turntable (because of the Centrifugal force), interesting trajectories will be observed while the rolling object is bearing on turntable without being expelled from it like charged particles absorbing in an electric field .Frequency of the turntable and rolling friction force are the most important parameters affecting on the trajectory of the rolling object. We need to know about rolling motion for our investigation. If an object rolls without slipping, then the bottom of the rolling object must be momentarily at rest. Rolling without slipping can be thought of as the motion of the center of mass plus rotational motion about its center of mass.

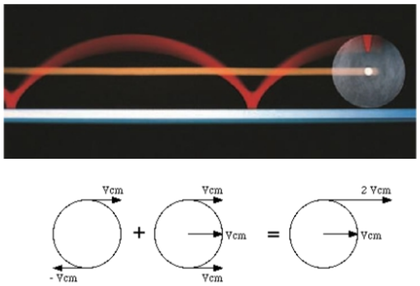


Fig. 1: Rolling motion of a rolling object

As shown in figure (1) when a rolling object has rolling motion it rotates about the point of contact with the ground.

Rolling friction is the resistive force that slows down the motion of a rolling ball or wheel. This type of friction is typically a combination of several friction forces at the point of contact between the wheel and the ground or other surface (Fig

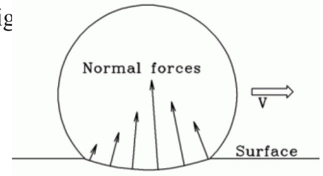


Fig. 2: Several friction forces

Amount of rolling friction is calculated by following equation:

$$F_r = u_r W$$

Where, F_r is the resistive force of rolling frictional force ,u_r is a constant, called the coefficient of rolling friction for the given surfaces of contact-which is dependent on elasticity and deformation amount of the surface- and W is the entire weight of the rolling object including the rolling mechanism.

In this problem rolling friction force plays a major role so discussion about effect of rolling friction is essential to investigate trajectories of spheres and rings on a rotating disc.

We investigated behaviors of spheres and rings (or discs) on the rotating disc separately.

2. Methodology

During our investigation we found articles which dealt with this problem using force, torque and angular motion equations and imaginary methods to make the problem understanding simpler. But our main way to solve the problem was experimental method. We produced a horizontal rotating disc and observed different types of trajectories for spheres and ring recording the rolling object motion by digital high speed camera. We get our experimental data using "Tracker" software (Fig. 3).

Fig. 3: Experimental setup

3. First Attempt: Different Observed Types of Motion for Spheres

We can easily observe these types of trajectories doing simple experiments through a turntable and a sphere.

1. The ball's center of mass goes in a curved path and expels from disc (Fig. 4a).

2. The ball's center of mass runs on the disc and additionally exhibits loops. (Fig. 4b)

3. The ball's center of mass turns in a stable circular orbit that depends on the frequency of the rotating disc.(Fig. 4c)

4. The ball's center of mass is at rest and has no motion.

Fig. 4: (a) The ball expells from disc. (b) The ball makes loops (c) The ball runs on a circular orbit

Fig. 5: The ball is being expelled from the disc

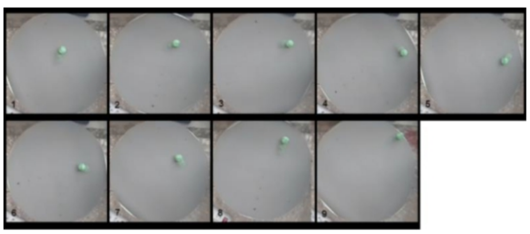


Fig. 6: The ball makes loops

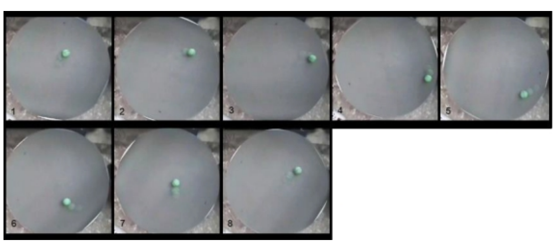


Fig.7: The ball turns in a circular orbit

4. Analyze of Trajectory of Spheres

Here r, R, ω and Ω are the sphere radius, the radial distance of the sphere from disc center, angular velocity of the sphere and angular velocity of the turntable respectively.

1. In this situation the linear velocity of the turntable is much more than the surface velocity of the sphere:

$$\omega\times r = v+\Omega\times R$$

2. friction force makes the ball roll on rotating disc and the ball has angular velocity perpendicular to the velocity of the disc and it keeps its rolling direction while the direction of the velocity of the rotating disc keeps changing but the disc can't change the rolling direction of ball (because of the angular inertia of the ball) so ball's contact point runs on the disc-we have sliding motion additionally-as a result the ball get closer to the disc but the surface velocity of the ball is less than the linear velocity of the turntable($\omega\times r = v+\Omega\times R$) so the ball gets to be expelled from the disc but friction makes ball's surface velocity increase and the ball repeat the trajectory back to back.

3. The ball keeps its velocity (because of its angular inertia) while it's coming toward the center so when it gets closer to the center its surface velocity is more than the linear velocity of the turntable at the point of contact. As a result it moves in opposite direction with its last movement and makes a circular orbit.

According to our experiments: $\omega_0=(2.7)\,\omega_d$

Where ω_0 is the frequency of the circular orbit and ω_d is the frequency of the rotating disc.

4. To keep sphere's center of mass at rest (have no velocity relative to outward coordinate system) its surface velocity must be equal to turntable velocity:

$$r\omega = R\times\Omega$$

V. Second attempt: motion of rings

There is no complete written theory for the trajectory of rings and discs in references so we started an investigation to analyze motion of them.

Rings and balls have various propinquities but rings because of the fulcrum have restriction against rolling with some of directions so the rotation of the turntable effects on the ring differently and makes it rotate around its diameter and with direction perpendicular to the turntable surface.

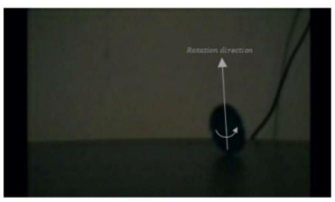


Fig. 8: There is rotation around the diameter of rings and changing mass center velocity direction during motion of rings

Discs and rings show .approximately the same interesting trajectories but inertia and angular inertia are actually important which is different for rings and discs. In figures (9) and (10) you can see the trajectory of a ring on the turntable.

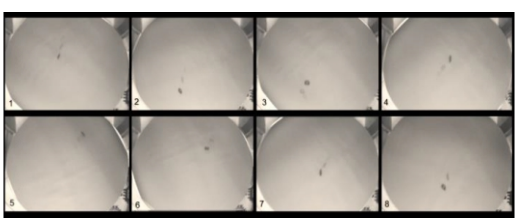


Fig. 9:Trajectory of a ring on rotating disc

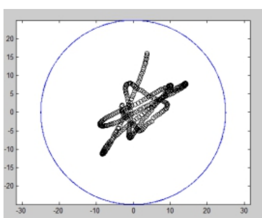


Fig.10: Trajectory of a ring on the turntable

5. Result

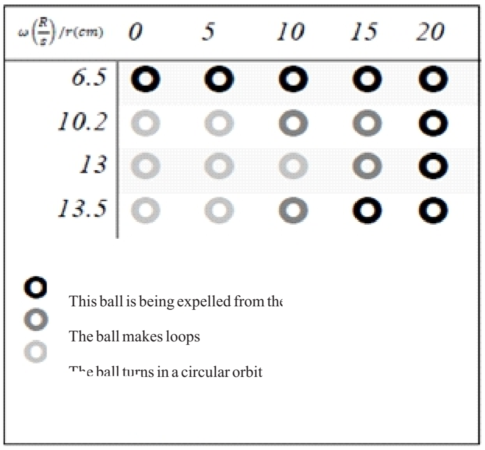
In figure (8) you can see the different trajectories of the ball on the turntable. We changed initial conditions and achieved table (1).

As shown by figure (8) the trajectory of the ball is dependent on initial conditions and the range of observation for a trajectory is limited to specific initial situation.

6. Conclusion

A sphere or a ring (or other type of rolling object) can bear on a rotating disc without being expelled from the disc exhibiting different types of trajectories that depends on initial conditions like frequency of the turntable and radial position of the rolling object and ratio of initial velocity of the rolling object into the velocity of the rotating disc and the range of observation for a trajectory is limited to specific initial situation. Type of friction affecting on the rolling object is different for different situations. (For example rolling friction for rolling motion and both of rolling friction and sliding friction for motion of a rolling object with sliding motion additionally).

Table 1: observed trajectories during experiments in different initial conditions



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