LIGHT RINGS

Seyed Ali Madani, Muhamad Hassan Muhammadi

ARTICLE INFO

Participated in PYPT

Accepted by Ariaian Young Innovative Minds Institute , AYIMI

http://www.ayimi.org ,info@ayimi.org

ABSTRACT

When an obstacle is put in the path of liquid jet, while jet meets some specifications, standing waves are created upstream of the obstacle. If the contact point of jet and obstacle is illuminated by laser beams, those standing waves act like an optical fiber and keep the light inside the jet. Laser beams after multiple internal reflections inside the jet, create rings of light around the jet. In this paper, first we investigate the phenomena in terms of fluid mechanics, determining the wavelength of standing waves and, as the second part, we examine the rings in terms of optics.

Keywords : *Liquid jet, Optical fiber, Fluid mechanics, Rings*

1. Introduction

Collision of the jet with an obstacle, cause standing waves upstream of the obstacle. This phenomena can be observed in low velocity jets of household tap. About this phenomena there aren't much papers in relation to an analogue phenomena called capillary waves on fluid cylindrical. The analyses about capillary waves, deal with the instability that cause the cylindrical eventually become droplets. The first person who investigated this phenomena was Rayleigh which dated back to last century. He assumed waves of form $e^{-i(kz-\omega t)}$ on a jet with radius r and velocity u in the direction z with surface tension σ and density ρ and wave number k , he obtained this equation

$$\omega^2 = \frac{\sigma I_1(kr_0)}{\rho r_0^3 I_0(kr_0)}(kr_0)(k^2r_0^2 - 1)$$

In most of the papers which are published about standing waves in fluid cylindrical, the base of the analyses is the equation above. In these papers, the effect of viscosity is and also we face with lack of experimental data in this field.

In this article, we first investigate the phenomena without involving viscosity. We determine the effect of different parameters on these waves, which may haven't been investigated in any papers and then, we let the viscosity come in play and investigate standing waves, on highly viscous threads.

2. Theory

In order to analyze what physical effects are causing this phenomena, we should find out the behaviors of a liquid jet. First we examine the shape of a liquid jet.

2.1. The Shape of a Falling Fluid filaments (jet)

We investigate the shape of jets with high Re numbers, which the effects of viscosity on the shape and movement of the jet is negligible. Using Bernoulli's Equation for points A and B, we'll have :

$$1/2\, \rho v_0{}^2 + \rho g l + P_A = 1/2\, \rho v^2 + P_B$$

According to Young-Laplace Equation that describes the capillary pressure difference sustained across the interface between two static fluids, due to surface tension, where a R_1 and R_2 are the radius of local curvature of jet:

$$\Delta \mathrm{p} = \sigma \nabla . \boldsymbol{n} = \sigma \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

That we'll get:

$$\nabla . \boldsymbol{n} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) \approx \frac{1}{r}$$

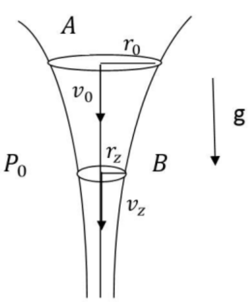

Fig. 1: The fluid cylindrical, accelerating under influence of gravity and surface tension.

Thus, with that simplification, the pressures of jet in points A and B are related to P_0 in this way:

$$P_A = P_0 + \frac{\sigma}{a}$$

$$P_B = P_0 + \frac{\sigma}{r}$$

Substituting into Bernoulli's equation and simplifications:

$$\frac{v_z}{v_0} = \sqrt{1 + \frac{2gr_0z}{v_0{}^2} + \frac{2\sigma}{\rho v_0{}^2 r_0}\left(1 - \frac{r_0}{r}\right)}$$

Now, we have the equation for the instantaneous velocity of the jet, we can easily obtain the equation for the radius of the jet using flux conservation:

$$Q = 2\pi \int_0^r v_z r_z dr = \pi r^2 v_z = \pi r_0^2 v_0$$

Hence we'll this equation:

$$\frac{r}{r_0} = \sqrt{\frac{v_0}{v_z}}$$

From this equation we can see that the jet will become narrow as it get more speed under influence of gravity.

2.2. Plateau–Rayleigh Instability

When a cylindrical jet flows in contact with air, there are some perturbations on the surface of the jet. These perturbations are always present and can be generated by numerous sources including vibrations of the fluid container or non-uniformity in the shear stress on the free surface. These disturbances form arbitrary. In fact, when a curve is made in the surface of the thread, because the surface tension acts in such a way to have the minimum interface with the gas phase (in order to reach the minimum potential energy) the surface will push back against any curvature to make the possible smoothest interface (mathematical proof that smooth shapes minimize surface area relies on use of the Euler–Lagrange equation). With the jet being pushed and then pulled back, we have a wave like shape.

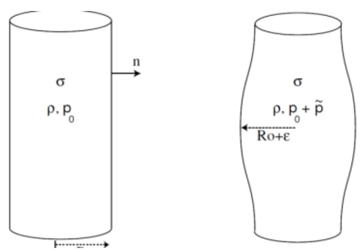

Fig.2: A cylindrical column of initial radius R_0 is shown in Steady State and Perturbed State

The radius of the stream R_1 is smaller , hence according to the young Laplace equation the pressure due to the surface tension is increased. Likewise at the peak the radius of the stream is greater and , by the same reasoning pressure due to the surface tension is reduced. If this was the only effect, we would expect that the higher pressure would squeeze liquid into the lower pressure region in the peak. In this way we see how the wave grows in amplitude over time.

But these two effects, don't cancel each other, as long as the effect of R_2 dominates the effect of R_2 the wave decays over time but in opposite situations, the wave grows over time. It is found that unstable components (that is, components that grow over time) are only those where the product of the wave number with the initial radius is less than unity.

$kR_0 < 1$

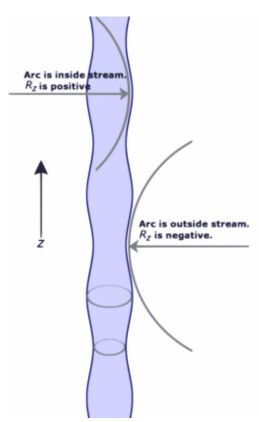

Fig. 3: Perturbations along a cylindrical jet

3. Main Explanation

Now, we investigate these waves in presence of an obstacle which is being pushed along the cylinder from one end. That growing wave will be compressed in length. By shifting the frame of reference, obstruction is moving upstream in the shifted frame of reference, that obstruction affects only the disturbed cylindrical surface. The jet itself continues to flow through it. It might be more helpful to say that we are witnessing the foreshortened history of the early, not-fully-developed, waves and again by changing the frame of reference and stand on the jet, those travel up above the obstacle with the same velocity as the jet which is falling in the exact opposite direction. Hence, those waves are stationary are observed like standing waves.

4. Materials and Methods

In our experimental study, we investigate effects of parameters on standing waves and confirm our main explanation due to our experimental results.

By changing the velocity we found that the wavelength of standing waves significantly is affected by velocity of the jet .

Increasing the velocity cause momentum increase. Hence our developing waves are more compressed in high velocity. More the Plateau Rayleigh are compressed, less the wavelength of standing waves will be. So, in high velocities we have standing waves with small wavelengths.

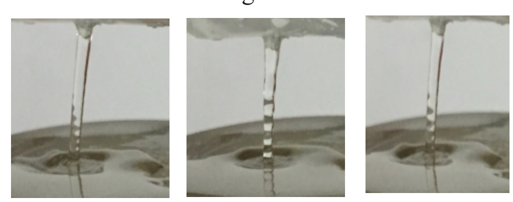

Fig. 4: Wavelength of standing waves is a function of jet velocity. Figure shows different velocities with the same radius

In case of investigating how does radius of the jet affect on standing waves, we canceled the effect of radius on velocity of the jet by experimenting different radiuses under the same velocity. Effect of radius on standing waves is even more than the effect of velocity. Because changing radius except affecting on the wavelength, will increase the velocity which itself affects on waves.

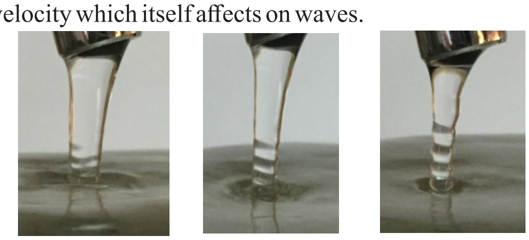

Fig. 5: Increasing the initial radius of the jet, causes wavelength of standing waves being decreased

A surprising result is that the wavelength of standing waves is independent of radius in high velocities. The most important parameter which affect on standing waves is surface tension of jet in contact with air. Again in changing the surface tension of the jet, we considered that surface tension will affect on jet velocity. In order to cancel this effect, we experimented different surface tensions under same velocities.

Surface tension is the main reason of this phenomena and is the most effective parameter. Due to our theory, increasing surface tension cause Plateau Rayleigh waves being increased. Hence, wavelength of standing waves are bigger in higher surface tensions of jet. Another surprising result is that in high velocities, wavelength of standing waves is independent of jet surface tension.

5. Conclusion

Using our experimental results, we found out how wavelength of standing waves changes under different velocities and radiuses of the jet. We also investigated the main parameter, surface tension. We figured out a significant change in wavelength of standing waves by changing the surface tension, which proves our theory that is based on capillary actions. These waves, act as an optical fiber which keeps light inside.

FUTURE WORK

As our future works, first we have plan on investigating this phenomena in presence of viscosity using Navier Stokes equations.

Also, we have plan on experimental study of viscous threads, Using Capillary number to state a limit between capillary actions and viscosity effects which eventually will tell us in what capillary numbers, this phenomena occurs .

References

[1] K. M. Awati and T. Howes. Stationary waves on cylindrical fluid jets. Am. J. Phys. 64, 808(1996)

[2] T. Massalha and R. M. Digilov. The shape function of a free-falling laminar jet: Makinguse of Bernoulli's equation. Am. J. Phys. 81, 733 (2013)

[3] S. Senchenko and T. Bohr. Shape and stability of a viscous thread. Phys. Rev. E 71,056301 (2005)

[4] S. L. Goren and S. Wronski. The shape of low-speed capillary jets of Newtonian liquids. J. Fluid Mech. 25, 01, 185-198 (1966)

[5] liquid jet over a horizontal plane. J. Fluid Mech. 20,03, 481-499 (1964)

[6] Rayleigh L. XVI. On the instability of a cylinder of viscous liquid under capillary force. Philos Mag 1892;34.

[7] GAVIS, J. 1964 Contribution of surface tension to expansion and contraction of capillary jets. Phys. Fluids, 7, 1097.