# Geometry and Dynamics of the Fractal Fingers

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A B S T R A C T

Fractal fingering is a phenomenon that occurs when a droplet of an ink-alcohol mixture is deposited onto diluted acrylic paint. This effect characterized by the formation of intricate, branching patterns that resemble fractal. The geometry and dynamics of these fingers are influenced by several parameters, including the viscosity and surface tension of the fluids, as well as the concentration and composition of the ink-alcohol mixture. In this paper, we investigate the effect of these parameters on the formation and evolution of fractal fingers in order to gain a better understanding of this fascinating phenomenon.

***Keywords :*** *: Fractal , Acrylic, fluid, , Ink Alcohol mixture*

## 1. Introduction

Fractal fingering is a phenomenon that can be observed in many different contexts, from the formation of river deltas to the growth of crystals. The geometry and dynamics of the fingers that form during this process are influenced by a number of relevant parameters, including the viscosity and surface tension of the fluids involved, as well as the rate at which they are mixed.

One important factor that affects fractal fingering is the viscosity of the fluids involved. When two fluids with different viscosities come into contact, they tend to mix in a way that creates complex patterns. In the case of fractal fingering, this mixing result in the formation of thin, branching fingers that extend outwards from the droplet.

Another important parameter is surface tension. When two fluids with different surface tension come into contact, they tend to form patterns that reflect relative strengths. In the case of fractal fingering, this can result in complex patterns that resemble tree branches or lightning bolts. The rate at which the fluids are mixed also plays a role in determining the geometry and dynamics of fractal fingers. If one fluid is added slowly to another, for example, it may be possible to observe more intricate patterns than if both fluids are mixed quickly.

Overall, understanding how fractal fingering works requires a deep understanding of fluid dynamics and interfacial phenomenon. By studying these processes in detail, physicist can gain insights into how complex patterns emerge from seemingly simple interactions between fluids. This knowledge has applications in fields ranging from materials science to environmental engineering, and could ultimately help us better understand and control natural phenomena like river deltas or crystal growth.

## 2. Materials and Methods

Ink as a solvent, soluble alcohol and diluted acrylic are used in this research to investigate the behavior of fractal. Their required properties are listed in Tables 1, 2, and 3.

We considered diluted acrylic as a fluid with a higher viscosity in a glass plate as a base fluid and drop a droplet of ink-alcohol mixture and investigated the behavior of the fractal. No external pressure gradients or injections were applied to drive the flow. All experiments were conducted at ambient temperatures. A camera (Canon EOS 800D) was used to capture fractal behavior directly. They were recorded at 60/50 fps.

**Table 1:** Ink- Alcohol different concentration viscosity

| | | | | |
|---|---|---|---|---|
| Alcohol(ml) | 4.5(25%) | 9(50%) | 13.5(75%) | 16.2(90%) |
| Ink(ml) | 13.5(75%) | 9(50%) | 4.5(25%) | 1.8(10%) |
| Viscosity(pa.s) | 0.003475 | 0.002517 | 0.001885 | 0.001582 |

**Table 2:** Diluted acrylic viscosity

| | | | | |
|---|---|---|---|---|
| Acrylic(ml) | 16.2(90%) | 13.5(75%) | 9(50%) | 4.5(25%) |
| Water(ml) | 1.8(10%) | 4.5(25%) | 9(50%) | 13.5(75%) |
| Viscosity(pa.s) | 0.248936 | 0.121658 | 0.06090 | 0.013591 |

**Table 3:** Diluted acrylic surface tension

| | | | | |
|---|---|---|---|---|
| Acrylic(ml) | 16.2(90%) | 13.5(75%) | 9(50%) | 4.5(25%) |
| Water(ml) | 1.8(10%) | 4.5(25%) | 9(50%) | 13.5(75%) |
| Viscosity(N/m) | - | 0.0035 | 0.02 | 0.0067 |

## 3. Experiments and Results

### 3.1. Spontaneous Three-Stage Mixing

The results obtained from repeated experiments show that the formation of fractal consists of three separate phases.

After the drop of alcohol-alcohol hits the diluted acrylic surface, the first phase occurs; In this phase, the ink-alcohol begins to diffuse on the diluted acrylic surface and after that the second phase begins; In the second phase, fractal shapes begin to form. Shapes like tree branches, each of which divides into two branches that are half of the previous one, and they compete with each other to divide more. In the third phase, which lasts longer than the

the previous phases, the formed fractal patterns diffuse and dissolve after a long time, and we will not have a fractal pattern anymore.

### 3.2. Dependence on Ink-Alcohol Composition

The degree of fractal fingering, was found to be related to the composition of the ink-alcohol. By using different compositions of alcohol-ink in our experiments, we found out that when we change the percentage of our mixture, their physical properties change, especially the viscosity (Fig.1), which affects this phenomenon, and finally, the change in viscosity affects the shape and the way fractal are formed. The effect of ink-alcohol viscosity is as follows:

With the increase of alcohol in our mixture, its viscosity also decreases; By testing different percentages of alcohol and ink, we can conclude that the higher the amount of alcohol (at constant surface tension), the finer and narrower the fractal will be. (Figs.2 and 3).

### 3.3. Dependence on Diluted Acrylic Composition

Almost like the previous part, by changing the composition of diluted acrylic, its physical properties change. The physical properties that affect the phenomenon of diluted acrylic are surface tension and viscosity. As mentioned before, with the increase in viscosity (Fig.4), the fractal becomes smaller and narrower; But when we consider the ink-alcohol mixture constant and work with it on different diluted acrylic compositions, the result will be that with the increase in surface tension (which occurs due to the increase in water), the fractal in a constant percentage of Ink-alcohol becomes wider. (Figs. 5 and 6).

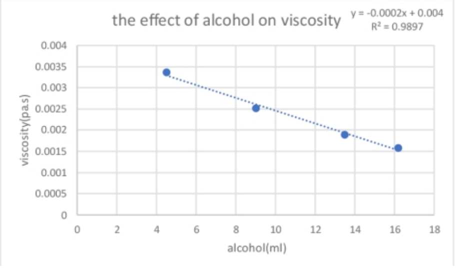


**Fig. 1:** The effect of alcohol on viscosity

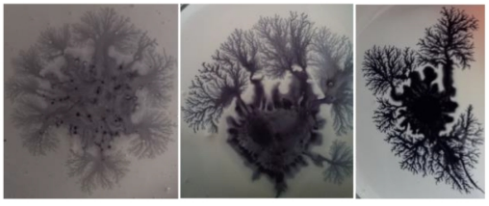

**Fig. 2:** Different compositions of alcohol on constants surface tension of 25% water

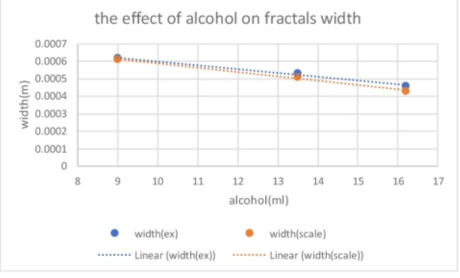


**Fig. 3:** The effect of alcohol on fractal width

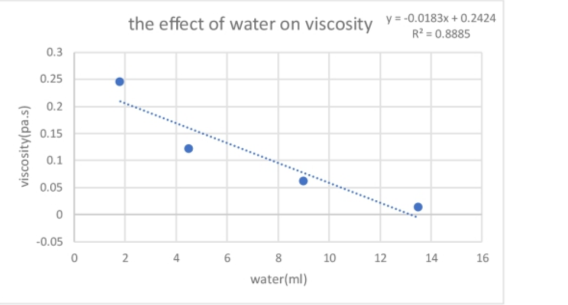


**Fig. 4:** The effect of water on viscosity

**Fig. 5:** Considering the constant of ink-alcohol compositions and the increase of surface tension, we see that the width of the fractal has become wider

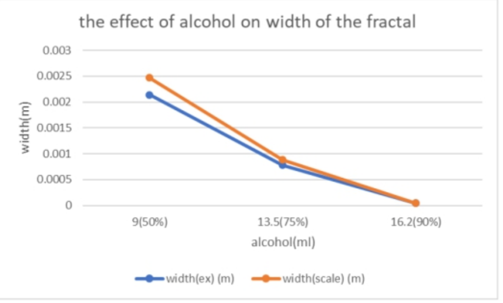


**Fig. 6:** By increasing the surface tension and testing the same compositions as before, it can be found that the width of the fractals has become wider

### 3.4. Velocity

The velocity of formation of fractal, as well as how it is formed, depends on parameters such as viscosity and surface tension of mixtures. The result we got from repeated experiments is that by keeping the diluted acrylic concentration constant (keeping the surface tension constant) and changing and increasing the ink-alcohol composition, the velocity of forming fractal increases (Fig. 7).

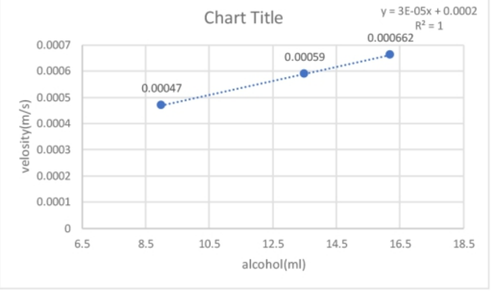


**Fig. 7:** Constant surface tension with 25% water and different ink-alcohol compositions that led to increased velocity

Also, if we keep the same ink-alcohol compositions and dilute the acrylic (increase the surface tension), the velocity of forming fractal will increase (Fig. 8).

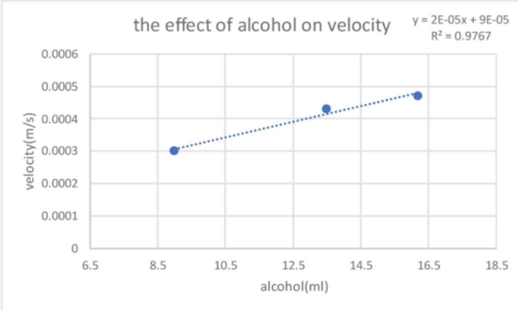


**Fig. 8:** As the surface tension increases, the speed also increases

## 4. Discussion

In the previous parts, we observed that without any external pressure, fractal fingers begin to form spontaneously when we drop a droplet of ink-alcohol mixture on diluted acrylic.

Our goal was to investigate the geometry and dynamics of these fractal patterns, which we reached many results after research.

### 4.1. Three-Stage Spontaneous Fractal Fingering Due to Multicomponent Character of Diluted Acrylic

Under no external pressure gradients, a surprising three-stage diffusive process was observed wherein ink-alcohol appears to finger spontaneously into diluted acrylic.

Acrylic itself is composed of various components such as polymer, pigment, etc.; This is the reason why acrylic has components of different sizes and it can be considered a multi-component solution. In diluted acrylic, small, lighter components are mobile and are susceptible to diffusion. Heavy, branched components, on the other hand, are less mobile in comparison and their self-affinity retains the heavy components in the original diluted acrylic phase. Similarly, lighter components enable the diffusion of other species, such as the ink-alcohol, to diffusive easily, whereas heavier components retard the diffusion of such solvents. As a result, when in contact with a miscible solvent such as ink-alcohol mixture, differential rates of diffusion arise due to molecular size and mobility**.** The fractal-like fingering process continues until light, mobile components are extracted preferentially by diffusion. Full extraction of the light components marks the conclusion of spontaneous fingering in first stage and the onset of slow diffusion in second stage.

The spontaneous fingers that formed in this phenomenon exhibit self-similarity .

### 4.2. Existence of Distinct Interfaces

In pressure-driven Saffman-Taylor fingering, instabilities are driven by the most unstable wavelength, λc, given by:

$$\lambda_c = \pi b \sqrt{\frac{\sigma}{\Delta\eta V}}$$

where $b$ is the height of the ink-alcohol mixture, $\sigma$ is the surface tension, $\Delta\eta$ is the difference in viscosity between diluted acrylic and ink-alcohol, and $V$ is the interfacial velocity. Considering that different compositions were used for the experiments in this article, values should be obtained for each of the above variables with any series of experiments.

The capillary formula mentioned above is for the conditions of Saffman-Taylor fingering experiments, and in order to apply it to our experimental conditions, we must consider $b$ something else.

In the condition of their experiment a liquid was placed between two glass sheets and another liquid injected on it with a syringe attached to it (Fig. 9); in that situation $b$ was the gap between two glasses but in the context of our problem $b$ should be considered as something else. When we drop a droplet of ink-alcohol onto diluted acrylic a new surface is formed on our previous surface (Fig. 10); $b$ should be considered as the height of the new surface (Fig. 11).

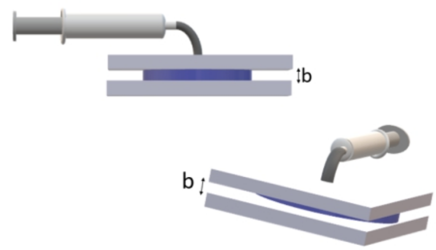


**Fig.9:** Simplified model of Hele-shaw cell

**Fig. 10:** ink-alcohol onto diluted acrylic

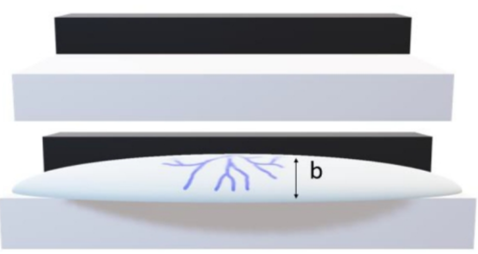


**Fig.11:** The height of the ink-alcohol mixture

## 5. Conclusion

The three-stage spontaneous dispersive mechanism observed reveals a surprising mechanism that underlies the Saffman-Taylor instability for miscible, multicomponent fluids. In this work, spontaneous fractal fingering between Ink-alcohol mixture and diluted acrylic were studied.

To be sure of the obtained results and their accuracy, we compared the width of the fractal obtained through our experiments with the widths obtained through the scales taken from the photos of the experiments and converted into real numbers and units (Table 2 and 3).

In this comparison, it can be seen that the obtained data are slightly different from the real data, and this proves to us that the results obtained through these researches and experiments were without problems.

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