FLOATING OF A METAL DISK ON THE WATER SURFACE

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ABSTRACT

When a metal disk is placed in a vessel of water it shall sink due to its higher density but the water jet impinges on the center of the disk may cause it to be floated . The hydraulic jump is formed in the edges of the disk by the water jet that passes radially away from the center of the disk to the edges. This jump causes a difference in the level of the height above and under the disk which will increases pressure under the disk. Water above the disk is moving with higher velocity compared its velocity to water under the disk so due to Bernoulli's principle, the pressure difference between them increases. pressure difference and then minimum velocity of the water jet.

Keywords : *Metal disk, Hydraulic jump, Water jet, Stability*

1. Introduction

In this research a metal disk in a vessel of water is studied. A metal disk with a hole at its center sinks in a container filled with water. when a vertical water jet hits the center of the disk , it may float on the surface. It shall sink due to its higher density but as an external force here the water jet impinges on the center of the disk and it is observed that the disk floats.To find relevant parameters several experiments are designed with qualitative analysis.

2. Theories and Methods

In the beginning there are two possibilities:

1- The radius of the water jet is smaller than the radius of the whole so the whole flow of the water jet passes through the hole.

2- In the second scenario radius of the water jet is bigger than the radius of the hole so that there is a flux of water passing radially away from the center.

Since the first case will not keep every metal disk floated we will observe the second scenario in more details (Figs.1 & 2).

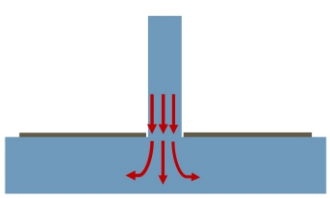


Fig. 1: The whole flow of water passes through the hole

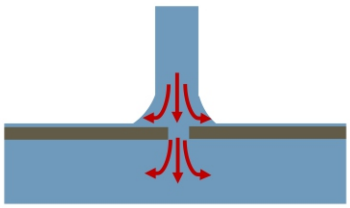


Fig. 2: Part of water flows radially away from the center to edges on the disk

To illustrate forces acting on our disk there is a free body diagram. Gravity force is pulling our disk downward then as the water jet impinges to the center there is an extra force acting downward but still we see our disk stays floated so there must be an upward force acting against our downward forces that prevent our disk from sinking now we want to find the origin and minimum amount of this upward force (Fig. 3).

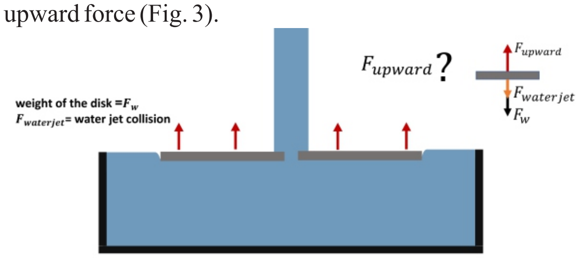


Fig. 3: Forces are applied on the disk

Before moving on to the theoretical framework there are some concepts that we need to understand. First hydraulic jump which as our jet impinges to the center of the disk it flows radially away from the center to the edges with higher velocity and makes the super critical flow then at the edges of our disk there is a jump formed with this super critical flow which collides with the subcritical flow with less velocity. To observe characteristics of this jump we defined two Froude number one refers to outer film depth being outer Froude number and one refers to inner film depth as inner Froude number (Figs. 4 &5) (Eqs. 1-3).

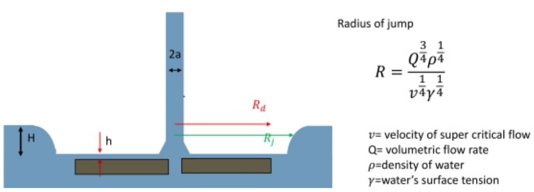


Fig. 4: Hydraulic jump and its radius

Froude number :

$$F_{ro} = \frac{Q}{2\pi R_j\sqrt{gH^3}} \tag{1}$$

$$F_{ri} = \frac{\lambda Q}{2\pi R_j\sqrt{gh^3}} \tag{2}$$

Fr_o= outer Froude number based on outer film depth
Fr_i= Froude number based on inner film depth

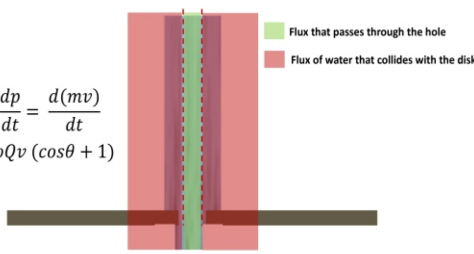
$$H = \left(\frac{-1 + \sqrt{1 + 8Fr_i^2}}{2} \right) h \tag{3}$$

Since the velocity of the thin layer of water above the disk is not uniform and constant we have a correction factor to be calculated also the height of the jump can be measured by using inner Froude number and depth of water inside the jump. There is a circular hydraulic jump in this phenomenon, radius of the jump can also be calculated proportional to velocity of super critical flow, volumetric flow rate, density of water and surface tension (Eq. 4) .

Since system is in equilibrium $\Longrightarrow \Sigma F = 0$

$$F_{downward} = F_{mg} + F_j$$
$$F_{up} = F_p$$
$$F_p = F_w + F_j$$
$$F_j = \rho Q v$$
$$F_w = mg \tag{4}$$

By Newton's second law and equation of momentum the water jet force is calculated but we have to note that the volumetric flow rate is not the whole water coming out of water jet. To have a better approach, there is an approximate illustration of the water jet . Two fluxes in the edges represented by red collide with the disk and the flux of water in the middle represented by green passes from the center of the hole (Fig. 5) (Eqs. 5&6).



$$F = \frac{dp}{dt} = \frac{d(mv)}{dt} \tag{5}$$

$$F_j = \rho Q v \,(cos\theta + 1) \tag{6}$$

Fig. 5: Flux passes through or collide the disk

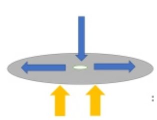
The other concept is Bernoulli's principle, when the water jet impinges to the center there is a part of water moving above the disk with higher velocity compared to the velocity of water under the disk leads to a difference in pressure above and under it and by higher pressure under the disk there is an upward force (Eq. 7).

Dynamic Pressure

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2 + Lp \tag{7}$$

Static Pressure

$v_1 > v_2$, $P_1 < P_2$ } Higher pressure under the disk $\longrightarrow F_{upward}$



Since our system is in equilibrium sum of all forces should equal to zero , downward forces of weight of the disk and water jet force. Upward force is caused by the difference in pressure which minimum amount of this force can be measured . On the other hand, there is another way to calculate this upward force which is multiplying the difference in pressure to the area of the disk in order to do that we have to find the difference in pressure above and under our disk (Fig.6) (Eq.8).

Using Bernoulli's principle that p_1 stands for pressure under the disk and p_2 stands for pressure above it and since the velocity of water under the disk is almost zero that component will be omitted also the height of water above the disk is negligible so that component would also be omitted lastly we can find the difference in pressure as this equation.

Pressure under the disk

$$F_p = mg + \rho Q v \; \} \quad p_1 + \cancel{0} + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + 0 \tag{8}$$

Pressure above the disk

on the other hand $F_p = \Delta PA$

Therefore : $\Delta P = \frac{1}{2}\rho v^2 - \rho g h$

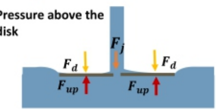


Fig. 6: Forces applied on the disk

Thickness of the thin layer of water above the disk should be measured to find characteristics of hydraulic jump and due to Watson's determination we defined Reynold's number and proportion of radius of the disk to the radius of the water jet and found this equation to calculate the thickness of thin layer but we still have to find the velocity of non-uniform film of water above the disk (Eq.9) (Fig. 7) .

$$\left.\begin{array}{l} Re = \frac{2Q}{\pi a v} \\ \frac{R_d}{a} = 0.366 Re^{\frac{1}{3}} \end{array}\right\} h = \frac{2\pi^2}{3\sqrt{3}} \frac{v(R_j^3 + 0.286a^3 Re)}{QR_j} \tag{9}$$

R_d = *radius of the disk*
a= radius of the jet
v = velocity of jet

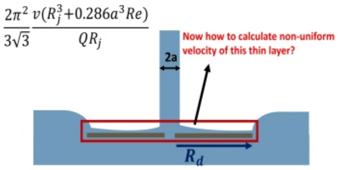


Fig. 7: Non-uniform velocity of the thin layer

Going back to the inner Froude number that we defined here we need to find the correction factor which by integrating from the velocity of the thin layer of water the equation above and by applying the boundary conditions final we can define the Froude number and due to Watson's solution we defined a delta here which represent the proportion of vertical location relative to the surface which Watson gave a solution to find the correction factor (Eqs.10-13).

$$F_{ri} = \frac{\lambda Q}{2\pi R_j \sqrt{gh^3}} \quad (\lambda \text{ is the correction factor }) \tag{10}$$

Inner Froude number can be written as :

$$F_{ri} = \frac{\sqrt{\frac{1}{h}\int_0^h u^2 dz}}{\sqrt{gh}} \tag{11}$$

If $u = f u_{z=h}$, $R_j > R_d$ and $Q = 2\pi R_j \int_0^h u dz$

$$\boldsymbol{F_{ri} = (\frac{\sqrt{\int_0^1 f^2 d\delta}}{\int_0^1 f d\delta}) \frac{Q}{2\pi R_j \sqrt{gh^3}}} \tag{12}$$

u= velocity of thin film
Z= vertical location relative to surface

$\delta = \frac{z}{h} = 1$ (free-surface of inner film)

Watson also suggested a solution $\Longrightarrow c\delta = \int_0^f (1 - x^3)^{-0.5} dx$

$\Longrightarrow$ c = 1.4

If $R_j > R_d$; $f(1) = 1$, $f(0) = 0$ (*boundary condition*)

The correction factor :

$$\lambda = \frac{\sqrt{\int_0^1 f^2 d\delta}}{\int_0^1 f \, d\delta} = \frac{3c^{\frac{3}{2}}}{\sqrt{2}\pi} = 1.12 \tag{13}$$

3. Experiment

A supply tank with an adjustable nozzle was used and water jet impinged to the center of various disks which for experimental data we varied radius of the water jet, radius of the hole, mass of the disk, radius of the disk and also the existence of hole in the center and for all experiments distance between nozzle and disk were constant .

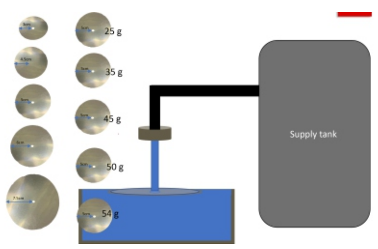


Fig. 8: Experimental Setup

4. Results

By analyzing the data it is observed as the radius of the disk increases the minimum velocity that was necessary for the disk to stay floated would also increase similarly. As the disk gets heavier minimum velocity of the water jet would also increase (Figs. 9&10).

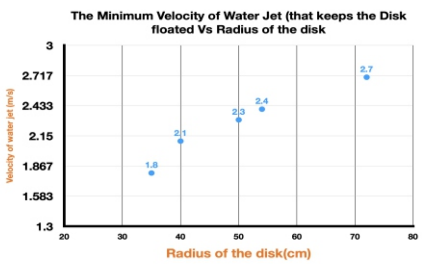


Fig. 9: The velocity of water jet versus radius of the disk

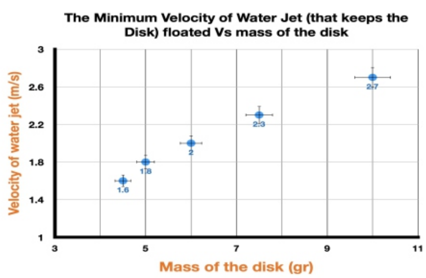


Fig. 10: The velocity of water jet versus mass of the disk

A color-coded chart is used to find how the proportion of radius of the disk and radius of the hole affect on the phenomenon qualitatively which the best floating was possible when the radius of water jet was bigger than the radius of the hole (Fig.11).

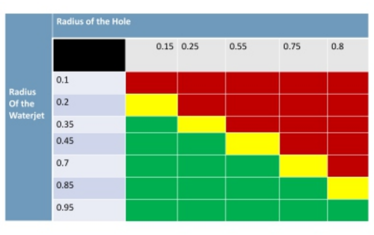


Fig. 11: Color-coded chart in analyzing data

Another color-coded chart is used to show how increasing each parameter changes the floating of the disk which green represents stable floating, yellow stands for floating with oscillation and red stands for sinking (Fig. 12).

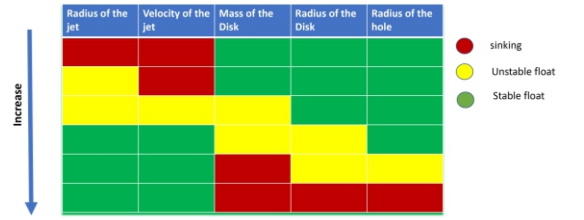


Fig. 12: Another color-coded chart in analyzing data

The next thing in our experiment was simulating the disk and impingement of water jet on the disk with and without the hole. In both cases hydraulic jump was observed but what is the reason that we don't see floating of a disk without the hole in the center (Fig. 13)?

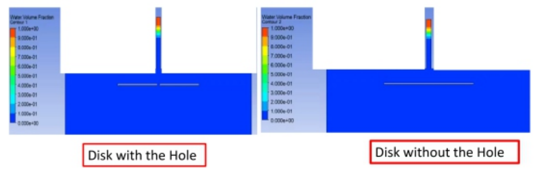


Fig. 13: Simulation results

we even tried to investigate this experimental that the disk with a dent in the center is floated in stable position but for the disk without the hole floating was possible in the beginning but within less than a second it started sinking. Because when the water jet is applied to a disk without any dent on the surface the reaction force is vertically upward and since the disk is placed in water it can move horizontally away but for the second case that has a dent in the center , the reaction force from the edges of that dent can be divided to its x and y components which x components cancel each other and that is the reason that our water jet stays in the center and floating is stable.

By comparing the theory with experimental data we found the force caused by the difference in pressure in two ways and by putting them as equals, also the minimum velocity of the water jet that each disk needs in order to be floated .

5. Conclusions

The relation between minimum velocity and radius/mass of the disk also the radius of the hole in the center of the disk are found by the experiments.

The fact that upward force which is keeping our disk floated is mainly due to Bernoulli and hydraulic jump. The hole in the center of the disk works as a system stabilizer.

In order to see this phenomenon, radius of the water jet must be bigger than the radius of the hole in the center. The first scenario that we suggested only works for very light disks. By equations minimum velocity of the water jet that keeps our disk floated was calculated too.

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