THE PROBABILITY OF A CYLINDRICAL DICE TO BE LUCKY

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ABSTRACT

A coin is a circular disk so when its flipped it lands on either its heads or tails but technically it's really thin cylinder so there's a tiny chance for it to land on its edge. Dice are generally used to generate a random outcome in which the physical design and quantity of the dice thrown determines the mathematical outcome used mostly for different games all around the world but in this research we are going to find the most important factors in a cylindrical dice when we flip it to land on each one of the faces.

Keywords : *Cylindrical Dice, Probability, Mass Theory*

1. Introduction

The problem states that to land a coin on its side is often associated with the idea of a rare occurrence. What should be the physical and geometrical characteristics of a cylindrical dice so that it has the same probability to land on its side and one of its faces? (Fig.1).

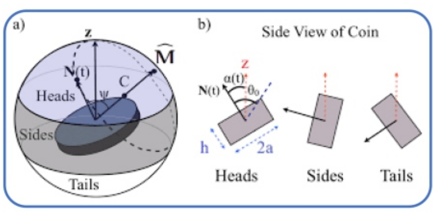


Fig.1: Paper spiral suspended above a candle

A coin is a circular disk so when its flipped it lands on either its heads or tails but technically it's really thin cylinder so there's a tiny chance for it to land on its edge but that's such a small probability that it doesn't even matter but there's also something like a pen which is the other extreme a very long cylinder if we flip it, it would land on its edge and if the pen was to be a coin the chance of the landing on its faces would be so little. So somewhere between the coin and the pen there's a cylindrical thick enough which makes it as likely to land on its edge and one of its faces and can be used as a three sided dice.

Dice is generally used to generate a random outcome in which the physical design and quantity of the dice thrown determines the mathematical outcome used mostly for different games all around the world. A cylinder on the other hand doesn't have an abundance of symmetry for various reasons least of it being the edge face doesn't have the same shape as the other two faces.

2. Theories and Methods

we inscribe the circular in a sphere. A cylindrical dice is made of two pieces; the smaller part is denser than the bigger one and they have equal masses, this moves the center of mass from the middle closer to the edge of the cylinder. The important way and variables such as height and angular velocity directly affect the throw's outcome. The number of throws is 1500 which the probability of getting an edge, P, is a third and 1-P is the probability of not getting an edge (2/3). Using these values, we can get a probability based on a normal approximation.

2.1. Center of Mass Theory

In this theory , considering that the center of mass is right in the middle of the cylinder , we divide the cylinder into 3 parts from the inside , each of which represents one of the faces . This theory states that all 3 parts should have the same mass (Fig. 2).

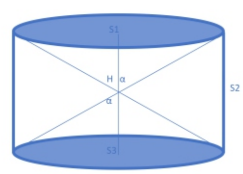


Fig.2: The cylinder is divided into 3 parts from the inside

2.2. Sphere theory

In throwing a sphere shaped ball the probability of it landing on any point on it is equal and this is where the idea comes from. This thick coin is embedded in a sphere where it fits the idea is that if we throw the sphere which has the thick coin inside of it one third of the area of the surface of the sphere is associated with one face one third a it is associated with the other face and the third associated with the band around the middle which is the edge (Fig.3) (Eqs. 1-4).

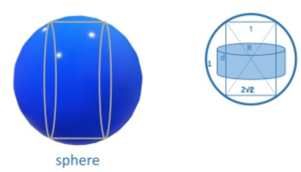


Fig.3: Thick coin is embedded in a sphere

$$S = 2\pi t R \quad (1)$$

$$S = \frac{4\pi R^2}{3}$$

$$t = \frac{2R}{3} \Rightarrow 2R = 3t \quad (2)$$

$$D^2 + t^2 = (2R)^2 = (3t)^2 \quad (3)$$

$$D^2 + t^2 = 9t^2 \quad (4)$$

$$D^2 = 8t^2 \qquad D = 2\sqrt{2}t$$

2.3. 2D Version

Now let's look at it from a 2d perspective. The coin is now fitted in a circle so taking the rectangular cross section of the disk and doing essentially the same technique but it 2d which means dividing the perimeter of the circle into third which means dividing the perimeter of the circle into thirds (Eqs. 5-7) (Fig. 4).

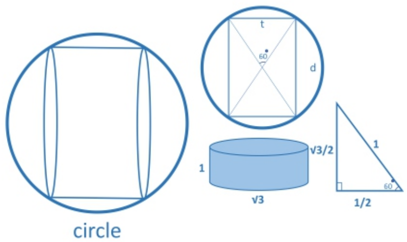


Fig. 4: The coin is now fitted in a circle

$$D = 2R\sin 60^0 = R\sqrt{3} \quad (5)$$

$$t = 2R\cos 60^0 = R \quad (6)$$

$$\boldsymbol{D = \sqrt{3}t} \quad (7)$$

So with the first 2 theories we found the shortest and the tallest extremes then with the third theory we got ratio diameter to thickness but they are both super theoretical and non-might be right so the only way to prove that they are right is an experiment repeated hundreds of times keeping all the factors except the ratio diameter to thickness constant in order to find the ideal ratio that gives the third probability .

3. Experiment

First I did 3D Printed Cylinders with ratio diameter thickness of route two and route 3 with the same mass and material.

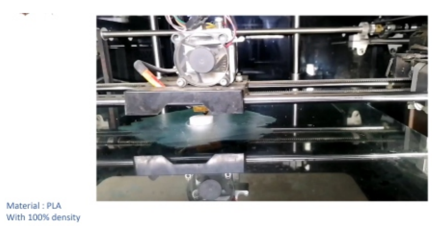


Fig. 5: 3D Printed Cylinders

Random Throw :then I figured out a way to a random throw . It's important the way the cylinders are thrown cause it has a direct effect on the outcome . The throw has to be random but not every single parameter has to be random in the throw . Variables like height and angular velocity directly affect on the throw's outcome . The angle which the cylinders are thrown at , is the random parameter in this method .

4. Statistical Analysis

Here we assume that the probability of a third landing on each side is correct with the given ratios and then calculate the likeliness of our data if that's true (specifically the edge). What is the chance of seeing 395 edges if it's got probability of third ?

we can calculate that using binomial distribution which says from our 1500 throws we got 395 edges if the probability of success (an edge) is a third what is the chance of that happening ? The answer is a very tiny probability but this isn't surprising considering that this is the probability of exactly 395 edges so what we do instead is saying what is the probability of getting less than or equal to 395 edges ; we'd need to calculate the probability of 1 + the probability of 2 + all the way to the probability of 395 which with the binomial distribution that would take lots of time and is unnecessary so instead we approximate it using the normal distribution . Our expectation is 500 if we're right (that's from our binomial distribution) . We also get our variance from binomial distribution (the number is 1500 that's how many throws , P is a third (probability of getting an edge) and 1-P is the probability of not getting an edge (2/3) (Fig. 6). We can use these numbers to see how wide this bell curve needs to be . Using these values we can get a probability based on a normal approximation .

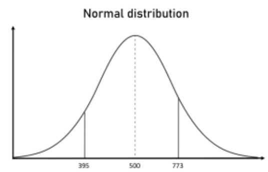
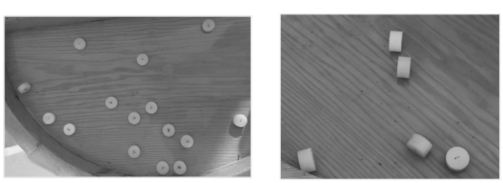


Fig. 6: Variance from binomial distribution

We got our average value here (this is what we would expect if it was a third) and we are saying what is the probability of 395 and how far away is it ? And it's still really tiny ; and even if we took a 2 sided test to see if it's in the extreme it would still be tiny . So we wouldn't have gotten these numbers if the ratios were correct . The probability of getting an edge is statistically different from a third with the 2 ratios. Although both the ratios were incorrect , they provided upper and lower bounds on the answer ; so I made cylinders with ratio diameter to thicknesses between 2√2 and √3 (started with the 2√2 ratio and made each cylinder 1 millimeter thicker to √3) (Fig.7a & b).



(a) (b)

Fig. 7: a)Thinnest cylinder 2√2 b) Thickest cylinder

From the chart the closest ratio to the third probability is 2.4. As shown the probability is a big number , and the difference from the third probability is less than 2% (Tables 1 &2).

Table 1: Ratio 2√2for total number of throws 1500

Number of	The results number	The expected number	The difference	Percentage
landing on one of its faces	1105	1000	105	73.66%
landing on its edge	395	500	-105	26.33%
landing on face 1	553	500	53	36.86%
landing on face 2	552	500	52	36.81%

Table 2: Ratio √3 for total number of throws 1500

Number of	The results number	The expected number	The difference	percentage
landing on one of its faces	727	1000	-273	48.47%
landing on its edge	773	500	273	51.53%
landing on face 1	351	500	-149	23.41%
landing on face 2	376	500	-124	25.06%

With an experiment like this it's about having the result close enough and 2.4 can be considered close enough but we can get even closer than that.

I made a cylindrical dice of two pieces; the smaller part is denser than the bigger one and they have equal masses, this moves the center of mass from the middle closer to the edge of the cylinder .This cylindrical dice is landed and the percentage of landing on the edge ,the heavier side, is more and this is where the idea of the mass theory comes from.

Then we experimented with 3D printed dice and the mass theories cylinders with ratio diameter to thicknesses of $2\sqrt{2}$, $\sqrt{3}$ and 2.4 with following results (Eqs. 8-13).

n=1500 P=1.3

Probability of Edge for $2\sqrt{2}$:

$$P(X=395)=\binom{1500}{395}(1.3)^{395}(2.3)^{1105}=1.78\times(10)^{-342} \quad (8)$$

Probability of Edge for $\sqrt{3}$:

$$P(X=773)=\binom{1500}{773}(1.3)^{773}(2.3)^{727}=7.42\times(10^{-267}) \quad (9)$$

$$P(X\leq 395)=P(X=1)+P(X=2)+\ldots+P(X=395)$$

$$P(X\geq 773)=P(X=773)+P(X=774)+\ldots+P(X=1500)$$

$$\emptyset\left(\frac{395-E(x)}{\sqrt{V(x)}}\right)\ E(x)=nP=500,\ V(x)=nP(1-P) \quad (10)$$

$$P(X\leq 395)=P(X=1)+P(X=2)+\ldots+P(X=395) \quad (11)$$

$$\emptyset\left(\frac{773-E(x)}{\sqrt{V(x)}}\right)\ E(x)=nP=500,\ V(x)=nP(1-P) \quad (12)$$

$$P(X\geq 773)=P(X=773)+P(X=774)+\ldots+P(X=1500) \quad (13)$$

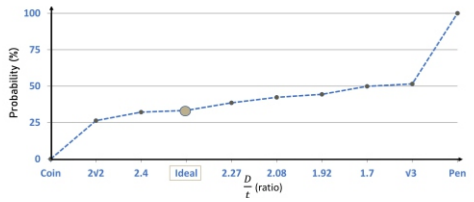


Fig.8: Probability of landing on the side

Table 3: Ratio 2.4 for total number of throws 600

Number of	The results number	The expected number	The difference	percentage
landing on one of its faces	407	400	7	67.83%
landing on its edge	193	200	-7	32.16%
landing on face 1	204	200	4	34%
landing on face 2	203	200	3	33.83%

They brought the results closer to the ideal so not only the geometric characteristics should be considered in this cylindrical dice but the physical properties play important role. There are lots of other physical properties that affects on the outcome of throwing variables like material, initial angular velocity, mass, the angle which the coin is thrown at, friction of the pieces, bounciness and more and the best way to study the effect of each of these variables and get closer than 1.57% to the third probability is a simulation (Fig. 9).

Finding how it works we throw thousands of cylinders while iterating over the cylinders thicknesses then find the thickness at which the coin is equally likely to land on each side then experiment is done a lot of times for different values of different physical properties to find how they affect on the outcome.

Fig.9: Simulation

5. Conclusions

The mass of the cylinder doesn't seem to make a big difference; it's pretty much steady at a thickness of 0.43 - 0.45 times the diameter. Initial angular velocity shows the more the cylinder spins in the air initially the thicker it has to be it increases between 0.42 and 0.46 (isn't a large difference) (Fig. 10).

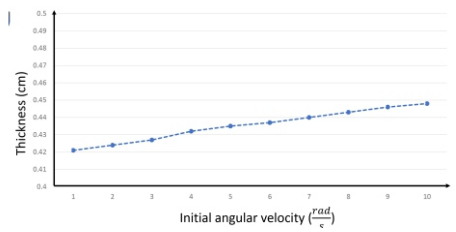


Fig.10: Thickness vs initial angular velocity of dice

The angle which the cylinder is thrown at the surface makes a huge difference; in this experiment it's the random parameter but I calculated the ratios with different given angles from 90 degrees to 270(Fig. 11).

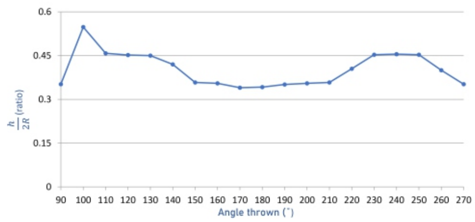


Fig.11: Angle of thrown effect on the results

Friction changes the results a lot and we need a much thicker coin when the coefficient of friction is between 0.1 and 0.18 but after that it decreases (Fig. 12).

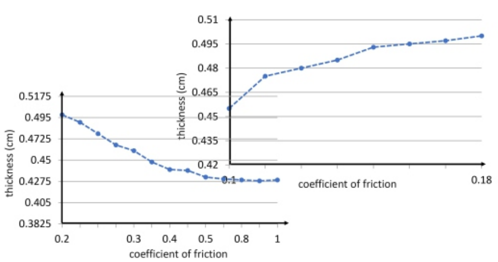


Fig.12: Friction changes on the results

After collecting data with the simulation the best ratio diameter to thickness was found which resulted in exactly third on each face lots of times and generally the probability of it landing on the edge is 2.4655 only 1.23%

further from 33.33% .

The 2.4655 ratio makes the closest to a perfect 3-sided dice but since this is a cylindrical and considering it doesn't have so many symmetry, with exaggerated changes on our physical properties the best cylindrical dice wouldn't be fair anymore so the best 3 sided fair dice would be a cube that has 2 obs 2 twos 2 thres on it.

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