Investigation of Looping Pendulum Movements

Rojan Abdollahzade Miralia , Tehran/ Iran, rojan.abzd@gmail.com

ARTICLE INFO

IYPT 2019 Iran Team member in Poland and Winner of Bronze Medal

Accepted in country selection by Ariaian Young Innovative Minds Institute , AYIMI

http://www.ayimi.org ,info@ayimi.org

ABSTRACT

Obviously, a pendulum is a very simple physical object that exhibits periodic movements. We present a physics-based approach to investigate characteristics of looping pendulum movements by several experiments and using tracker for the motion controllers to track in dynamics simulation. To control parameters our motion tracking adopts a method, which computes characters and captures data with real-time response in our experiments. The initial conditions is controlled to get the position of the path of both masses and the most important parameters radius and angle of the light mass and distance of the heavy mass from the origin are investigated.

Key Words *: pendulum , simulation, mass, radius, angle*

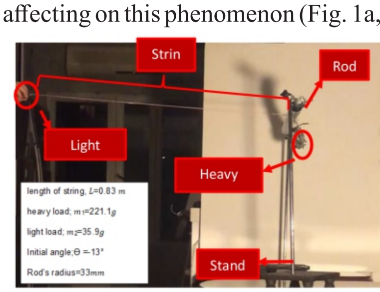
1 Introduction

A simple pendulum is composed of a weight, or bob, hanging freely from the end of a string or bar. Gravity pulls the bob in a downward arc, causing it to swing. There are different types of pendulum. Foucault pendulum swings in two dimensions is a type of simple pendulum, which demonstrates the rotation of the earth. Once the Foucault pendulum is set in motion, its swing will tend to rotate clockwise in a circle over the course of about a day and a half. Double pendulum, which is called a chaotic pendulum, consists of two simple pendulums, one suspended from the other. Double pendulums are used primarily in mathematical simulations. Conical pendulum, which was studied by Robert Hooke, is used to analyze the planets' orbital motions.

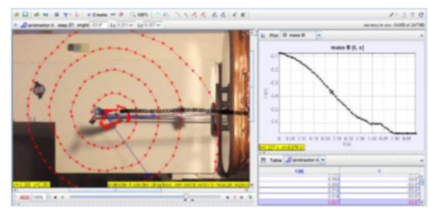
A looping pendulum consists of two loads, one heavy and one light, with a string over a horizontal rod. By pulling down the light one, the heavy load will lift up and by releasing the light load; it sweeps around the rod and keeps the heavy load from falling to the ground. If observers are able to use it, allow accurate estimation of looping pendulum parameters such as length or period in this study as linear functions or interpreting the results. Together with statements made during experiments we are showing that the rules are affecting the length is a linear function of 'speed', where speed appears to be a function of both period and angular velocity and so on.

2 Experimental Setup

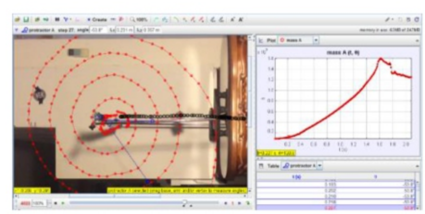
According to the structure of looping pendulum, our experimental setup was installed and then tracked the motion of the masses, to investigate the important parameters affecting on this phenomenon (Fig. 1a,b and c).



(a)



(b)



(c)

Fig. 1: a) Experimental setup; b) and c) tracking the motion of two masses A and B

In order to investigate the motion of the light mass we can define the vectors in the cross section of the rod $\hat{e}_r$ which is proportional to the radius of the rod and $\hat{e}_\theta$ which is in the direction of the string connected to the mass (Fig.2). For finding the position of the mass, we can use the $\vec{r}$ where it can be written as a function of the unit vectors (Eq. 1):

$$\vec{r} = R\hat{e}_r + x\hat{e}_\theta \quad (1)$$

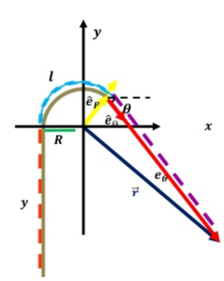


Fig. 2: Vectors in the cross section of the rod

3 Theories and Modeling

The acceleration of the mass is the second derivation of the vector $\vec{r}$ which consist of different terms of acceleration (Eq. 2).

$$\ddot{\vec{r}} = \vec{a} = -(2\dot{x}\dot{\theta} + R\dot{\theta}^2 + x\ddot{\theta})\hat{e}_r + (R\ddot{\theta} - x\dot{\theta}^2 + \ddot{x})\,\hat{e}_\theta \quad (2)$$

Centripetal, Linear, Coriolis, Tangential

We have four terms of acceleration in this system: Coriolis, centripetal, tangential and linear acceleration. By all the forces applying to the mass in the direction of $\hat{e}_\theta$ and $\hat{e}_r$ (Fig. 3); we can write the newton's second law of motion in both directions (Eq. 3 & 4).

$$\frac{-w_2 \cos\theta}{m_2} = -(\dot{x}\dot{\theta} + R\dot{\theta}^2 + x\ddot{\theta}) \quad (3)$$

$$\frac{w_2 \sin\theta - T_2}{m_2} = R\ddot{\theta} - x\dot{\theta}^2 + \ddot{x} \quad (4)$$

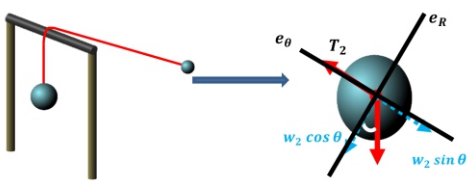


Fig. 3: The components of vectors on light mass

To solve these equations, the motion of the heavy mass in our system should be settled down.
Based on the behavior of the heavy mass we can divide the whole procedure into two phases:
1-The steady phase: both masses are moving and the string is sliding on the surface of riding, which means there is kinetic friction (Fig. 4a).
2-The motion phase: the heavy mass is at rest and we have static friction between the string and the rod (Fig.4b).

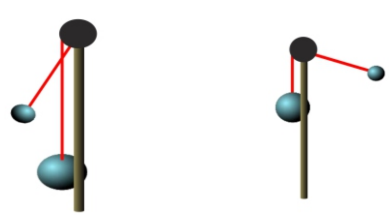


Fig. 4: a) The steady phase b) The sliding phase

The newton's second law is written for the heavy mass (Eq. 5 and 6) (Fig. 5):

$$w_1 - T_1 = m_1 a \quad (5)$$

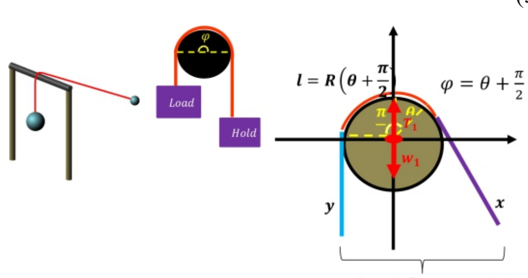


Fig. 5: The components of forces exert to the heavy mass

$$w_1 - T_1 = m_1(-\ddot{l} - \ddot{x}) \quad (6)$$

where the w_1 is the mass and T_1 is the tension force. The acceleration is found by assuming the string is always completely stretched without no extension so the length of the string remains constant. In sliding phase, we can write (Eq. 7, 8 and 9):

$$\frac{-w_2 \cos\theta}{m_2} = -(\dot{x}\dot{\theta} + R\dot{\theta}^2 + x\ddot{\theta}) \quad (7)$$

$$\frac{w_2 \sin\theta - T_2}{m_2} = R\ddot{\theta} - x\dot{\theta}^2 + \ddot{x} \quad (8)$$

$$l = R\left(\theta + \frac{\pi}{2}\right) \quad , \varphi = \theta + \frac{\pi}{2} \quad (9)$$

We can also relate the tension forces using the capstan equation (Eq. 10):

$$T_1 = T_2 e^{\mu\left(\theta + \frac{\pi}{2}\right)} \quad (10)$$

Using numerical model, these equations are solved for the sliding phase (Fig. 6).

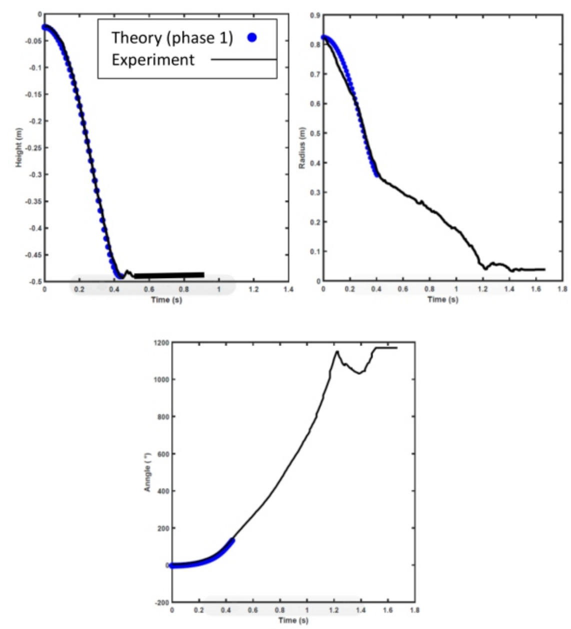


Fig. 6: The changes of height, radius and angle versus time in the sliding phase

L=0.83 m, $m1$=221.1g, $m2$=35.9g
Initial angle=-3°, Rod's radius=33mm
μ = 0.29

In the steady phase, we cannot use the capstan equation because there is no kinetic friction between the surface of the rod and the string. Since the heavy mass is at rest, the height of the heavy mass is constant and we will have the following equation (Eq. 11 and 12):

$$L = x + l + y, \ \dot{x} + \dot{l} = 0, \ \dot{x} = -\dot{l} \quad (11)$$

$$T_1 < T_2 e^{\mu\left(\theta + \frac{\pi}{2}\right)} \quad (12)$$

Now we can compare our theory and experimental results (Fig. 7).
The coefficient of friction is an important parameter and it can change the behavior of the masses and the whole function of the system. We have calculated the μ using two

different approaches; first by fitting the theoretical predictions and experimental results in different conditions and second by tracking the motion.

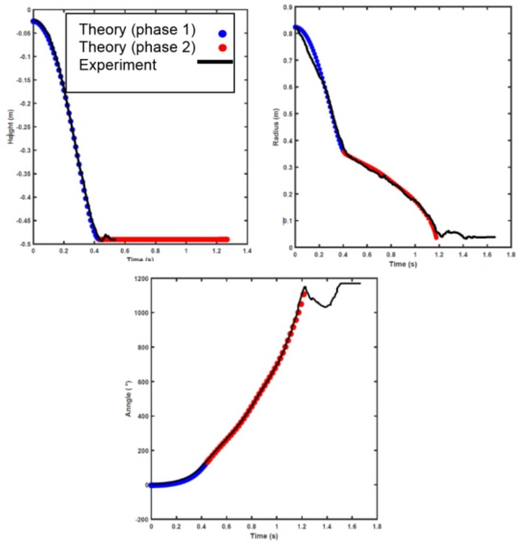


Fig. 7: The changes of height, radius and angle versus time in the steady state

L=0.83 m, $m1$=221.1g, $m2$=35.9g
Initial angle=-3°,Rod's radius=33mm
μ = 0.29

If we track the motion of a mass which is connected to the string and is sliding downward, we can find the coefficient of friction due to the acceleration of mass in different angles (Fig. 8).

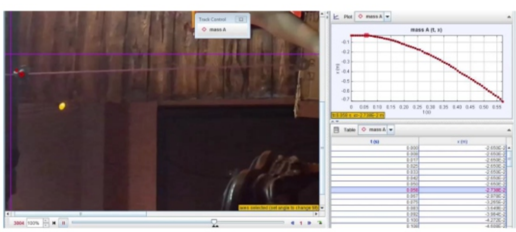


Fig. 8: Tracking the motion of mass to find the coefficient of friction

and by comparing the results we can see a great agreement (Eq. 13):

$$\mu_E = 0.31 \pm 0.03, \ \mu_f = 0.29, \ \mu_E \approx \mu_f \quad (13)$$

We have also investigated the effect of the μ on the total number of turns on the rod (Fig. 9).

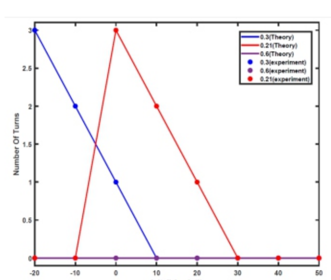


Fig. 9: Comparing the effect of μ in number of turns versus angle, experimental and theoretical

The path of the light mass is shown in figure (10) which is compared with the theoretical predictions.

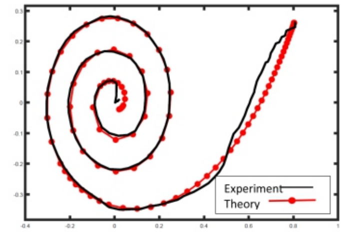


Fig. 10: The path of the light mass , comparing experiment and theory

The path in the second phase is an Archimedes spiral with the following equation:

$$y = bx\,sin(a + x) \quad (14)$$

where a defines the orientation and is proportional to the initial releasing angle and the b Which is the tightness and it depends on the radius of the rod.
Now we want to investigate the effect of the mass ratio on the motion on the heavy mass (Fig.11).

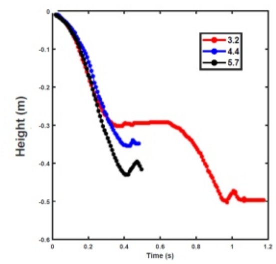


Fig. 11: The effect of the mass ratio on heavy mass

In figure (11), the height of the heavy mass is a function of time for different mass ratios. By decreasing the mass ratio, the minimum height will decrease and after a critical point, the heavy mass will move twice, after it comes at rest for the first time it starts moving downward again and it stops afterwards (Fig. 12).

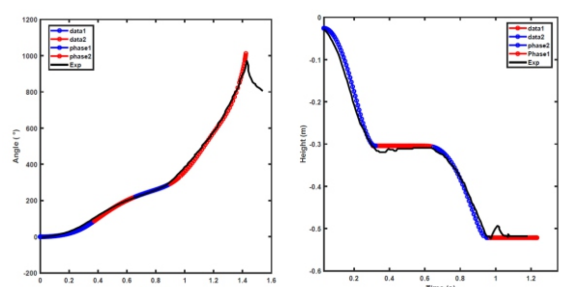
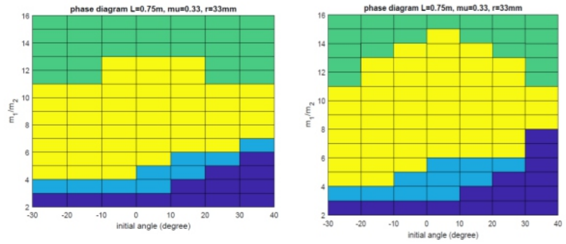


Fig. 12: . Returning Condition

The reason for the special returning condition is that when the light mass moves downward its acceleration will increase and the tension force will increase as a result so it will keep the heavy load. As the light load continues looping it will move upward and while the acceleration is decreasing the tension force will decrease too and the heavy mass will start moving again.

4 Discussion and conclusions

We have four different conditions for the system that under each circumstances we can observe a new behavior from the masses. Using the simulation results based on our theory we can compare the difference between experimental and theoretical results in different conditions (Fig. 13).



(a) (b)

Fig. 13: Comparing different conditions; a) Simulation, b) Experiment

Explanation of each color in figure (14);
- Dark blue: the initial conditions are not providing the needed energy for the first loop so the mass will fall off.
- Light blue: special returning condition happens and the heavy mass will be kept.
- Yellow: the light mass will loop around the rod and keep the heavy mass.
- Green: the light load will sweep around the rod and then will unloop and fall over the ground.

When the light mass comes at rest at the end, the magnitude of tension force will decrease and the load cannot keep the heavy load from falling anymore and this is the main reason for the special condition in green region in figure (14).

References

[1] D.Halliday , R.Resnick ,J.Walker. Fundamentals of Physics. John Wiley & Sons. (1923)

[2] https://sciencing.com/history-pendulum-4965313.html

[3] https://sciencedemonstrations.fas.harvard.edu/presentations/rope-friction-aroundpole

[4] wikipedia.org/wiki/Capstan_equation

[5] www.stevespanglerscience.com/lab/experiments/magic-pendulum/

[6] looping_pendulum_2_bil.pdf, https://www.istitutotrento5.it/images/test/bre_15_16