IRREVERSIBLE CARTESIAN DIVER

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ABSTRACT

This problem wants to investigate a simple Cartesian diver which is placed in a long vertical tube filled with water. Increasing the pressure in the tube forces the Cartesian diver to sink. When it reaches a certain depth, it never returns to the surface even if the pressure is changed back to its initial value. We are going to find how it depends on relevant parameters such as pressure, depth, initial velocity and other parameters.

Key Words: *Cartesian diver , Pressure, Depth, Velocity*

1 Introduction

The problem statement says " A simple Cartesian diver (e.g. an inverted test tube partially filled with water) is placed in a long vertical tube filled with water. Increasing the pressure in the tube forces the Cartesian diver to sink. When it reaches a certain depth, it never returns to the surface even if the pressure is changed back to its initial value. Investigate this phenomenon and how it depends on relevant parameters."

In our initial observation, when we squeeze the bottle, the diver sinks to the bottom of the bottle and after crossing a certain depth, it spontaneously sinks and doesn't come back to the surface when we decrease or release the pressure.

What happens to a simple Cartesian diver? The behavior of a Cartesian diver is unique and doesn't occur with normal buoys, but why?

When we squeezed the bottle, the air inside the diver compressed and water enters the diver. So part of the initial volume of the air that is inside the diver gets replaced by water. As we know, the density of water is much bigger than the density of air. Thus, the density of our diver increases, which causes our diver to sink.

When we release the pressure, water comes out of the diver. So the volume of the air increases and reaches its initial value. As a result, the density of our diver returns to its initial value. So our diver returns to its first position that was floating on the surface of the water.

The density of our diver changes due to the external pressure, so by increasing the pressure, the diver sinks. Now let's take a look at our diver. Here we have the initial length of air and water columns in our diver (Fig. 1).

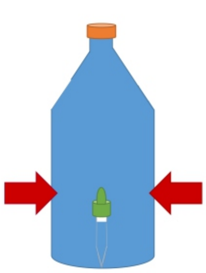


Fig.1: by increasing the pressure, the diver sinks

Air is compressible water is not. So by the rise of the total pressure, the air inside the diver compresses, and its volume decreases. So it allows more water to come inside of the diver. Thus the volume of water inside of the diver increases. As a result, the density of our diver rapidly increases, and it causes our diver to sink.

An irreversible Cartesian diver is a simple Cartesian diver with this difference; it remains at the bottom of the bottle, even when the pressure is released.

2 Theory and Methods

These are our assumptions:

- The air bubble in the diver feels only the ambient pressure plus the pressure exerted on the surface of water.
- The temperature of the water and the air trapped in the diver is constant during the experiment.
-Water is practically incompressible, so its density is constant and does not change with depth.
-The trapped air behaves like an ideal gas.

At first glance, we have the forces acting on our diver: The buoyant force, the diver's weight, and the dissipative force proportional to velocity (Fig. 2).

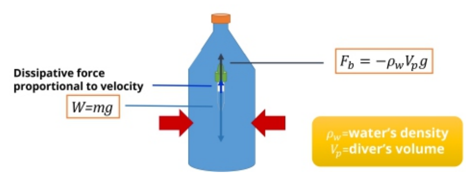


Fig. 2: Forces acting on the diver

Both of our theories show that we have a certain depth in a bottle of an irreversible Cartesian diver, which is called "Critical depth". When the diver goes beyond that depth, it spontaneously sinks and doesn't come back to the surface when the pressure is released.

But in a simple Cartesian diver, the bottle usually isn't long enough. So the diver always remains above that depth. So when it reaches the bottom of the bottle, the buoyant force will return it to the surface when we release the pressure. If we don't consider the drag force, we will realize that beyond the critical depth, the buoyant force that applied to our diver is lower than the diver's weight. So it diver at the bottom and the buoyant force won't be enough to overcome it and return the diver to the surface when the pressure is released (Fig. 3).

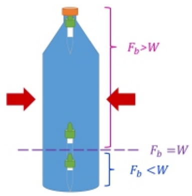


Fig. 3: The condition for never returning

So we wrote this equation of motion, without considering the drag force.

$$ma = mg - \rho_w V_p g \tag{1}$$

$$m\ddot{h} = mg - \frac{\rho_w nRTg}{P_a + \rho_w gh} \tag{2}$$

Now let's investigate the behavior of the diver in different pressures. Suppose we have this test tube, in P_0 pressure. The length of air column inside of it is l_0 and the length of the test tube is L. And suppose we have this bottle that the pressure of the air inside it is P. We place the test tube inside the bottle. The new length of air column in the diver is l, which is obviously smaller than l_0. We have x, that is the distance between the end of the diver and the free water level and α that is part of the of the air column in the diver that is submerged. We suppose our diver is the trapped air plus the test tube's glass (Fig. 4).

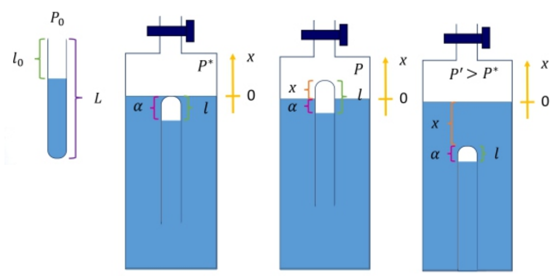


Fig. 4: The behavior of diver in different pressure

When we increase the pressure of the air inside of the bottle, x become lower until at a point, it equals to 0, at that point, the pressure is the "critical pressure"; that is defined as the pressure in which the end of the diver is at the same level as the liquid's surface. If we increase the pressure pass the critical pressure, our diver spontaneously and completely will sink.

Now let's discuss the first static equilibrium of our diver; when the pressure of the air inside of the bottle is P and in equilibrium position α is α_e (Fig. 5).

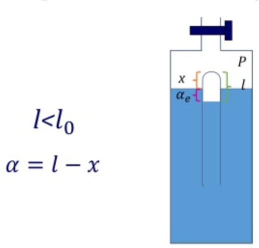


Fig. 5: The equilibrium position

The mass of the trapped air inside of the diver is negligible compared to the mass of the test tube so according to Archimedes' principle, we can rewrite this equation. The first term is the buoyant force exerted on air that is submerged and the second term is the buoyant force exerted on glass that is submerged.

$$mg = \underbrace{A\alpha_e\rho_w g}_{\text{Buoyant force exerted on air that is submerged.}} + \overbrace{V\left(1 - \frac{x_e}{L}\right)\rho_w g}^{\text{Buoyant force exerted on glass that is submerged.}} \tag{3}$$

Now we have the equilibrium equation when P is P^* and α_e^* is equal to l. So we can write this equation as equilibrium in critical pressure. This quantity depends only on geometrical parameters and on the two densities (Fig.6).

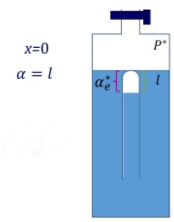


Fig. 6: Equilibrium with critical pressure

$$\boxed{\alpha_e^* = \frac{V}{A}\left(\frac{\rho_{glass}}{\rho_w} - 1\right) = L\left(\frac{d^2_{external}}{d^2_{internal}} - 1\right)\left(\frac{\rho_{glass}}{\rho_w} - 1\right)} \tag{4}$$

- $x_e \ll L \longrightarrow$ **We can ignore x_e**
- $\alpha_e \approx \alpha_e^*$

We suppose x_e is much smaller than L so we can neglect it and then α_e is approximately equal to α_e^* and we suppose the trapped air inside of the diver behaves like an ideal gas, and temperature is constant during the experiment (Fig. 7). So according to Pascal's principle and Boyle's law we can rewrite what we wrote in the previous step.

$$\boxed{\frac{P^*}{P_0} \approx \frac{l_0}{\alpha_e^*}} \tag{5}$$

$\alpha_e\rho_w g \ll P$ so it is neglected.

$$mg = V\rho_{glass}g = A\alpha_e\rho_w g + V\left(1 - \frac{x_e}{L}\right)\rho_w g \tag{6}$$

$$P_0 l_0 A = (P + \alpha_e\rho_w g)(\alpha_e + x_e)A \tag{7}$$

$$x_e = \frac{P_0 l_0}{P + \alpha_e\rho g} - \alpha_e \tag{8}$$

$$P_0 l_0 = (P^* + \alpha_e^*\rho_w g)\alpha_e^* \tag{9}$$

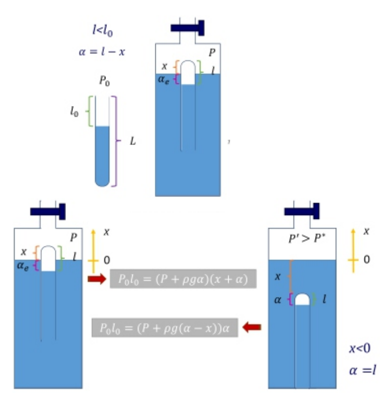


Fig. 7: First static equilibrium

Now let's take a look at the dynamics of our diver. In order to derive the equation of motion, we have to use Newton's second law. There are 4 forces acting on our diver: The buoyant force on air, the buoyant force on glass, the diver's weight and the dissipative force.

$$m\frac{d^2x}{dt^2}=\begin{cases} A\alpha(x)\rho_w g+V\left(1-\frac{x}{L}\right)\rho_w g-V\rho_{glass}g-b\dot{x}, & x>0 \\ \underbrace{A\alpha(x)\rho_w g}_{\text{Buoyant force exerted on air that is submerged.}}+\underbrace{V\rho_w g}_{\text{Buoyant force exerted on glass that is submerged.}}-\underbrace{V\rho_{glass}g}_{\text{Weight of the diver}}-\underbrace{b\dot{x}}_{\text{Dissipative force proportional to velocity}}, & x\le 0 \end{cases} \tag{10}$$

Now we have a differential equation should be solved to reach x(t), where b is the constant coefficient of velocity, and for a cylinder, C_D is 0.82, according to our reference.

$$b=\frac{1}{2}\rho_w C_D A \xrightarrow{\text{In a cylinder}} C_D=0.82$$

x_e is approximately equal to 0. So $\alpha(t)$ is smaller than α_e^* . The net force that is acting on the diver, might become negative and it causes the diver to sink.

$$\alpha(t)<\frac{V}{A}\left(\frac{\rho_{glass}}{\rho}-1\right)=\alpha_e^* \tag{11}$$

Our diver sinks when it reaches the no return depth. In that depth, x is equal to x_{nr} and α is equal to α_e^* . So x_{nr} is calculated

$$|x_{nr}|=\frac{1}{\rho g}\left(\frac{P_0 l_0}{\alpha_e^*}-P\right)-\alpha_e^* \Longrightarrow |x_{nr}|+\alpha_e^*=\frac{\Delta P}{\rho g} \tag{12}$$

By simulations and using tracker, the ordinary differential equation is solved So we used this library in python by the initial conditions: x and v_0 .

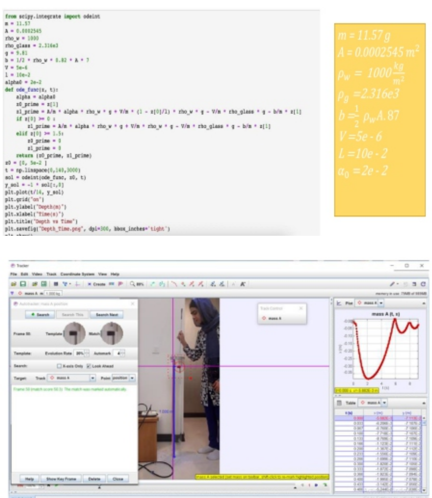


Fig. 8: using Simulation and tracker to solve equation

3 Experiments and Results

In this experimental setup, we used a cylindrical tube as the bottle, with 1 meter height and 10 centimeters diameter, a removable lead, at the top of the cylinder, a test tube, as the diver, that we marked it with black tape so that we are able to track it with Tracker, a pomp, to apply pressure on the liquid's surface, a three-way stopcock, to control the air admittance, a one-way stopcock, to control and maintain the pressure, a medical digital manometer and a ruler. So we are mechanically applying pressure on the water's surface and we are able to measure it.

Also several test tubes were used in our experiments (Fig. 9) .

Fig. 9: Experimental setup

In this experiment, we are displacing the diver from its first static equilibrium a little; it creates some damped oscillations in the diver, around its equilibrium point.

Fig 10: Damped oscillations of the diver around its equilibrium point

In other part of experiment, we have the irreversible Cartesian diver. When we increase the pressure, the diver sinks, even if we keep the pressure constant, when we reach P^* and when we release the pressure, the diver remains at the bottom of the cylinder.
Here we used several identical test tubes, with different l_0.

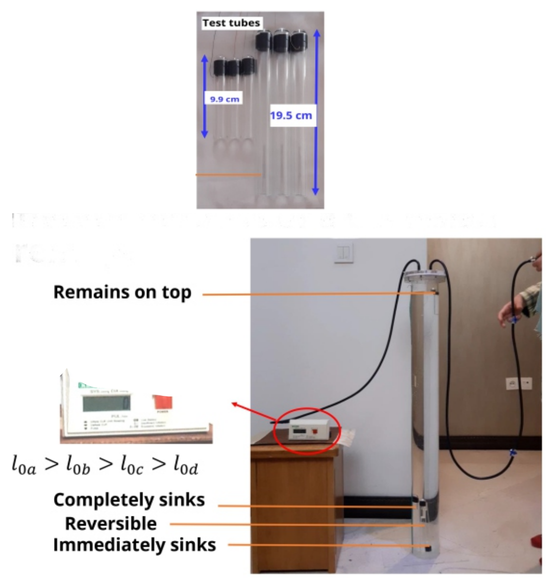


Fig. 11: Different motions of a Cartesian diver with different l_0

The diver d immediately sinks when we put it in the cylinder and when we increase the pressure, the diver c sinks. By increasing the pressure further, the diver b sinks too; because its critical pressure is lower than the diver a and higher than the diver c. But the diver a remains on top because its critical pressure is too much that we are not able to apply it. When we release the pressure, the diver b returns to the surface, but the positions of other divers won't change.

For the next experiment, we used one diver, with a specific l_0 but varying its initial velocities in constant pressure. Here are the graphs of the depths versus time when the initial velocities of our diver are :

$(v_1 < v_2 < v_3)$

With initial velocity v_1 , the diver goes down but because it doesn't reach the no return depth, it returns to the surface and has some damped oscillations around its equilibrium point.

v_2 is bigger, so the diver goes deeper but it doesn't sink completely and has a similar motion to the first situation, because it doesn't reach the no return depth either. But in the next situation, the diver crosses that depth and completely sinks and doesn't return to the surface. If we compare these graphs with those from the simulation, there is s good agreement between the theory and experiment (Fig. 12).

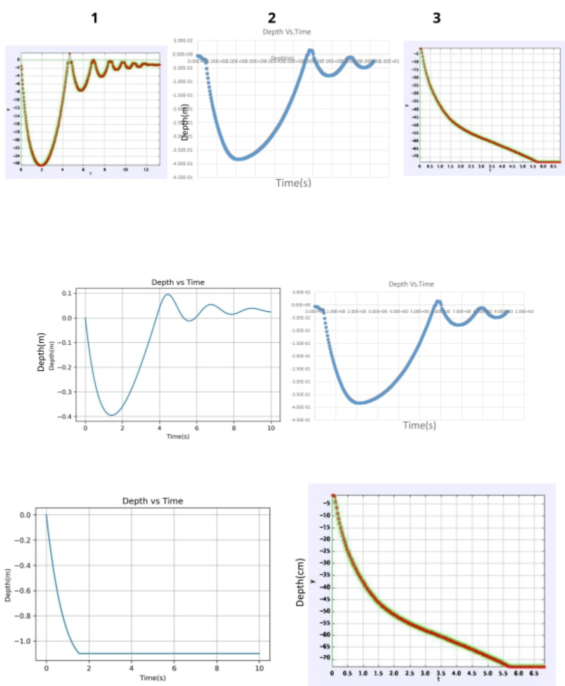


Fig 12: Different motions in the diver with different initial velocities

For the next experiment, we used one diver, but changing its l_0 and measuring the critical pressure for its amounts it is found they have a linear correlation (Fig. 13).

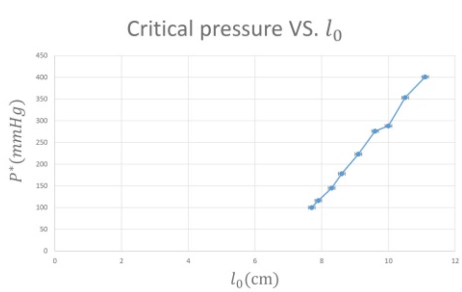
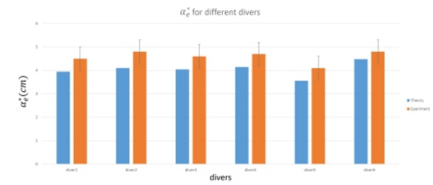
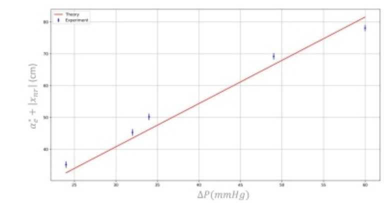


Fig 13: Linear relation in a diver between critical pressure and l_0

In comparison between the theory and the experiment for each of theses two α_e^* and $\alpha_e^* + |x_{nr}|$ we can find a good agreement (Fig. 14).





$$\frac{P^*}{P_0} \approx \frac{l_0}{\alpha_e^*}$$

	$l_0(cm)$	$\Delta P(mm\ Hg)$	$\frac{l_0}{\alpha_e^*}$	$\frac{P^*}{P_0}$
Diver1	4.5	24	1	1.316
Diver2	4.9	32	1.021	1.421
Diver3	4.9	49	1.065	1.645
Diver4	8	60	1.127	1.789
Diver5	8.1	34	1.110	1.447

Experiment
Theory

Fig 14: Comparison between theory and experiment

4 Conclusions

In this research we wanted to make a Cartesian diver that doesn't come back to the surface when the pressure is released, and to find the relevant parameters. Our theory said these following parameters are important and by adjusting them, we can have a Cartesian diver that when it goes bellow a certain depth, it never returns to the surface. But the irreversible Cartesian diver also can be achieved by some other ways, for example by using two liquids in the bottle with different densities, using absorbent material in the diver, etc.

When the external pressure is changed and the diver goes below a certain depth at the same time, we can have an irreversible Cartesian diver. That critical depth depends on the pressure and the initial amount of water in the diver.

When we give a small perturbation to a diver at its equilibrium point, it has some damped oscillations around its equilibrium point.

By using identical divers but with different initial conditions (l_0 , initial velocity, etc.) we can observe different motions.

The diver completely sinks when it reaches its no return depth and when the pressure is rose to the critical pressure it spontaneously sinks and also we observed a good agreement between the theory and the experiments.

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