1 Introduction
This experiment is about the motion of identical discs that are stacked on top of each other when we apply a sudden horizontal force to the bottom disc, the tower might remain standing.

When the sudden force is applied to the tower, other discs will start to move versus each other. If the center of mass of the discs passes through the last point of the bottom disc more than a critical time, the tower will topple because of the torque which is applied to the discs of the tower (Fig. 1).

Three kinds of movement have been observed in experimental results. The first kind was when we applied the sudden force to the bottom disc and the tower remain standing and also the movement between the discs of the tower is insignificant so we can consider it as a rigid body. The second kind is when the tower will remain standing but the movement between the discs of the tower is significant. This situation show the tower had the tendency to fall down and the third was when the tower topple (Fig. 2 a-c).

2 Theory and Experiment
In this paper, we are going to investigate in which cases the tower will remain standing and rigid, not rigid or it will fall.

2-1 Pattern of the Tower
When the sudden force is applied to the bottom disc the force will be transferred to the other discs as well but this transformation will happen with delay, so the first disc will start to move sooner than the other discs and it makes the distance between first discs to be shorter rather than the last discs as the pattern is observed in Figure (3).

2-2 Rigid Body Tower
We need to prevent this transformation with delay to have a rigid body tower. To avoid this transformation, the second disc must not start to move and accelerate. So the summation of the forces affecting the second disc must be zero (Fig. 4).
According to equations (1-3) the static friction affecting the disc from the upper discs and the kinetic friction from the lower disc must be equal to each other.

\[
f_{K_2} = mg\mu_k(n - 1) \tag{1}
\]

\[
f_{S_2} < mg\mu_k(n - 2) \tag{2}
\]

\[
f_{K_2} = f_{S_2} \tag{3}
\]

so:

\[
\frac{n - 1}{n - 2} < \frac{\mu_S}{\mu_K} \tag{4}
\]

By decreasing the amount of \( S \) the second disc will move less so the possibility of staying rigid will increase as well (Eq. 4). \( S \) have been investigated in different cases (Fig. 5).

![Fig. 5: Investigating the number of the discs](image)

As much as the number of the discs increases, \( S \) will decrease (Eq. 5). Also the inertia of the tower will increase as well so in high tower, hitting the bottom disc makes the angular acceleration of the tower less (Eq. 6).

\[
\frac{n - 1}{n - 2} < \frac{11}{10} < \frac{9}{8} < \frac{2}{1} < \frac{\mu_S}{\mu_K} \tag{5}
\]

\[
\tau = I\alpha \tag{6}
\]

![Fig. 6: The second disc doesn't move a lot because of the high number of the discs](image)

But when the number of the discs decrease, the pattern can be observed. According to Figure (7) the acceleration of the discs are equal to each other. So when the amount of \( S \) increases the tower will not stay rigid.

\[
\sum_{n=1}^{\infty} x > 0 \tag{7}
\]

If the summation of \( x \) is more than zero (that means the center of mass of the tower passes through the last point of the bottom disc is more than a critical time), the tower will topple when it is still over the bottom disc (Eq. 7).

![Fig. 7: Final pattern of the tower with low number of the discs](image)

\section{3 Effective Parameters}

Some effective parameters are introduced in this section. Also the effect of the parameters on possibility of toppling has been explained.

\subsection{3-1 Friction Coefficient between the Discs}

According to Fig. (11), the acceleration given to discs is equal to \( \mu_k g \).

When we increase \( \mu_k \) the acceleration given to each disc will increase. That makes the second disc to have a greater velocity. So the distance between the discs will increase and a not rigid pattern can be observed and this increases the possibility of toppling.
3-2 Mass of the Discs
From equation (7) we concluded when mass of each disc increases the inertia of the tower increase. So the angle of the tower doesn't change when it hits the bottom disc and the possibility of toppling decreases.

3-3 Radius of Each Disc
In cases that the radius of each disc is high, the tower will be stable. The duration that the center of mass of the tower passes through the last point of the bottom disc will less than a critical time. But when the radius decreases the center of mass passes through the red line sooner so the tower will topple (Fig. 12).

3-4 Speed of the Hand
When the speed of the hand is low, the duration that the tower and the bottom disc are touched with each other will increase (Eq. 8). So the second disc will have a greater velocity and not rigid pattern will be observed and that makes the tower to topple.

\[ \mu_k g \Delta T = v \]

(8)

4 Experimental Procedures

4-1 Number of the Discs and Radius of each Disc
Different number of the discs have been investigated (4 cm diameter) of each disc (Fig. 13).

As much as the number of the discs increases the S will decrease and it makes the tower to remain standing. It is also the same in figure (13) the possibility of staying rigid increases with number of the discs. When the radius of each disc increases, possibility of toppling increases with the number of the discs (Fig. 14).

4-2 Changing the Friction Coefficient
Some papers have been attached to the discs to change the friction coefficient. Therefore, when the friction coefficient is high the acceleration of the discs increases and the possibility of toppling increases as well and this matches the charts.

4-3 Height of Each Disc
Two discs have been attached to each other to increase the height of each disc. When the height of each disc increases, n will decrease so S decrease and the tower will topple (Fig. 15).

When the radius of the discs increases, the tower will have a stable situation so increasing the number of the discs will make it stable to a critical point. After that critical point, the effect of the speed of the hand will be important. When the speed of the hand is low the tower will topple because the second disc will have a high velocity otherwise the tower will remain standing.
In all, discs in 3cm height had a higher possibility of toppling rather than 1.5 cm.

5 Ratio
A variable has been defined to conclude the effect of height and radius of the discs.

\[
\text{ratio} = \frac{\text{height of the tower}}{\text{Diameter of the bottom disc}}
\]

It can be concluded that by increasing the ratio, the possibility of toppling increases as well but the slope is decreasing after a while (Fig. 16). That is because of the inertia. When the number of the discs is higher than a critical number, the inertia of the tower affects the tower a lot so the tower will not rotate when it hits the bottom disc. So the possibility of toppling will start to decrease again when the number of the discs increases.

6 Conclusion
Three kinds of movement have been observed in this phenomenal. Rigid body, not rigid body and toppling. The pattern of the tower has been explained. These kind of movement have been explained according to the theory. We found in which cases there is a rigid body or not rigid body and also in which cases the tower will topple.

Some affective parameters have been investigated theoretically and experimentally and possibility of toppling have been investigated too.

When we increase \( \mu_k \) the acceleration given to each disc will increase. That makes the second disc to have a greater velocity. So the distance between discs will increase and a not rigid pattern can be observed and this increases the possibility of toppling.

In cases that the radius of each disc is high, the tower will be stable. The duration that the center of mass of the tower passes through the last point of the bottom disc will be less than a critical time. But when the radius decreases the center of mass passes through the red line sooner so the tower will topple.

When the height of each disc increases, \( n \) will decrease in so \( S \) decreases and the tower will topple.

References