FALLING TOWER

Sahar Semsarha ,Farzanegan2 high school, Tehran/ Iran, sahar.sems@gmail.com

A B S T R A C T

This research is about the motion of identical discs that are stacked on top of each other when we apply a sudden horizontal force to the bottom disc, the tower might remain standing. We are going to investigate in which cases the tower will remain standing and rigid, not rigid or it will fall. Is it affected by the friction between the discs or other parameters?

Key Words *Tower, Discs, Force, Friction*

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Innovative Minds Institute , AYIMI

http://www.ayimi.org ,info@ayimi.org

1 Introduction

This experiment is about the motion of identical discs that are stacked on top of each other if we apply a sudden horizontal force to the bottom disc, the tower might remain standing.

When the sudden force is applied to the tower, other discs will start to move versus each other. If the center of mass of the discs passes through the last point of the bottom disc more than a critical time, the tower will topple because of the torque which is applied to the discs of the tower (Fig.1).

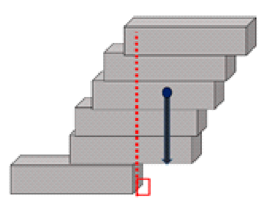


Fig. 1: Falling tower modeling (passing through the last point of the bottom disc)

Three kinds of movement have been observed in experimental results. The first kind was when we applied the sudden force to the bottom disc and the tower remain standing and also the movement between the discs of the tower is insignificant so we can consider it as a rigid body. The second kind is when the tower will remain standing but the movement between the discs of the tower is significant. This situation show the tower had the tendency to fall down and the third was when the tower topple (Fig. 2 a-c).

(a) (b) (c)

Fig. 2: a) Rigid body tower, b) Not Rigid body tower c) Falling tower

In this paper, we are going to investigate in which cases the tower will remain standing and rigid, not rigid or it will fall.

2 Theory and Experiment

In experimental setup, the sudden force has been applied to the bottom disc with a ruler and some wooden discs are used as tower. The thickness of the discs was 1.5cm with different diameters (4,6,8,10cm).

2-1 Pattern of the Tower

When the sudden force is applied to the bottom disc the force will be transferred to the other discs as well but this transformation will happen with delay, so the first disc will start to move sooner than the other discs and it makes the distance between first discs to be shorter rather than the last discs as the pattern is observed in Figure (3).

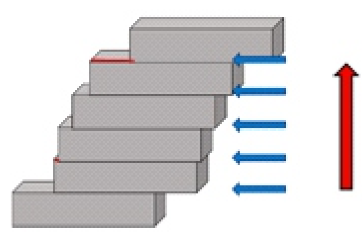


Fig. 3: Pattern of the tower after transformation

2-2 Rigid Body Tower

We need to prevent this transformation with delay to have a rigid body tower. To avoid this transformation, the second disc must not start to move and accelerate. So the summation of the forces affecting the second disc must be zero (Fig. 4).

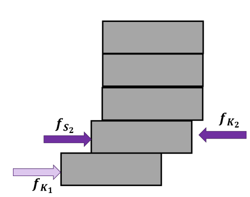


Fig. 4: Second disc free diagram

According to equations (1-3) the static friction affecting the disc from the upper discs and the kinetic friction from the lower disc must be equal to each other.

$$f_{K_2} = mg\mu_k(n-1) \tag{1}$$

$$f_{S_2} < mg\mu_S(n-2) \tag{2}$$

$$f_{K_2} = f_{S_2} \tag{3}$$

so:

$$\frac{n-1}{n-2} < \frac{\mu_S}{\mu_K} \tag{4}$$

By decreasing the amount of ***S*** the second disc will move less so the possibility of staying rigid will increase as well (Eq. 4). ***S*** have been investigated in different cases (Fig. 5).

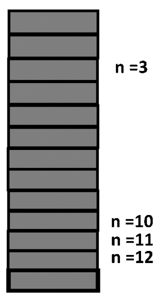


Fig. 5: Investigating the number of the discs

As much as the number of the discs increases , *S* will decrease (Eq. 5). Also the inertia of the tower will increase as well so in high tower, hitting the bottom disc makes the angular acceleration of the tower less (Eq.6).
So increasing the number of the discs increases the possibility of staying rigid as well (Fig. 6).

$$\frac{n-1}{n-2} \Rightarrow \frac{11}{10} < \frac{10}{9} < \frac{9}{8} < \frac{2}{1} < \frac{\mu_S}{\mu_K} \tag{5}$$

$$\tau = I\alpha \tag{6}$$

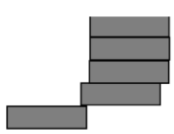


Fig. 6: The second disc doesn't move a lot because of the high number of the discs

But when the number of the discs decrease, the pattern can be observed. According to Figure (7) the acceleration of the discs are equal to each other. So when the amount of ***S*** increases the tower will not stay rigid.

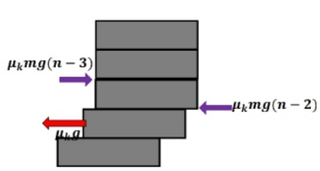


Fig. 7: Final pattern of the tower with low number of the discs

2-3 Falling Tower

The toppling will happen in not rigid cases after the pattern is observed. The center of mass of this pattern must be investigated.

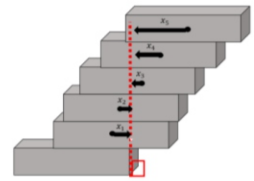


Fig. 8: Place of the center of mass

If the summation of x is more than zero (that means the center of mass of the tower passes through the last point of the bottom disc is more than a critical time), the tower will topple when it is still over the bottom disc (Eq.7).

$$\frac{\sum_2^n x}{n-1} > 0 \tag{7}$$

When the tower is coming down, it will hit the bottom disc for a short time This phenomenon will change the angel of the tower and it makes the torque to be applied to the disc of the tower. Torque makes the discs of the tower to move versus each other so the place of the center of mass must be investigated again.
If the equation (7) become more than zero in figure (9) the tower will topple in the moment that it hits the ground. Otherwise the tower will remain standing in a not rigid pattern (Fig. 10).

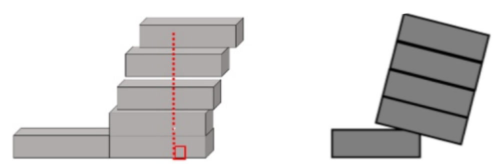


Fig. 9 : Changing the angle of the tower

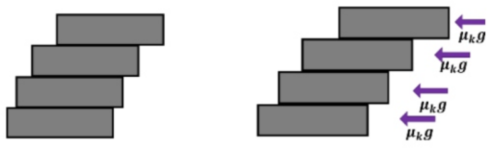
Fig.10: not rigid tower

3 Effective Parameters

Some effective parameters are introduced in this section. also the effect of the parameters on possibility of toppling has been explained.

3-1 Friction Coefficient between the Discs

According to Fig. (11), the acceleration given to discs is equal to $\mu_k g$.
When we increase μ_k the acceleration given to each disc will increase. That makes the second disc to have a greater velocity. So the distance between the discs will increase and a not rigid pattern can be observed and this increases the possibility of toppling.



(a) (b)

Fig. 11: a) Lower friction modeling, b)Higher friction modeling

3-2 Mass of the Discs

From equation (7) we concluded when mass of each disc increases the inertia of the tower increase. So the angle of the tower doesn't change when it hits the bottom disc and the possibility of toppling decreases.

3-3 Radius of Each Disc

In cases that the radius of each disc is high, the tower will be stable. The duration that the center of mass of the tower passes through the last point of the bottom disc will less than a critical time. But when the radius decreases the center of mass passes through the red line sooner so the tower will topple (Fig. 12).

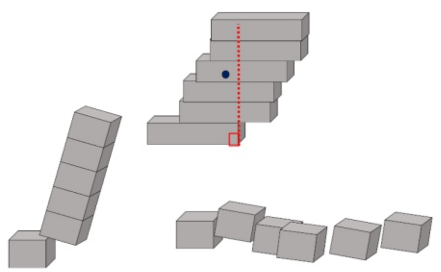


Fig. 12: Different radius of the discs

3-4 Speed of the Hand

When the speed of the hand is low, the duration that the tower and the bottom disc are touched with each other will increase (Eq. 8). So the second disc will have a greater velocity and not rigid pattern will be observed and that makes the tower to topple.

$$\mu_k g \Delta T = v \tag{8}$$

4 Experimental Procedures

4-1 Number of the Discs and Radius of each Disc

Different number of the discs have been investigated (4 cm diameter) of each disc (Fig. 13).

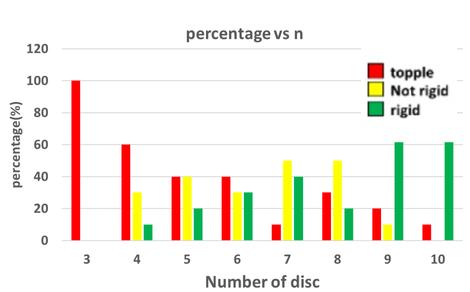
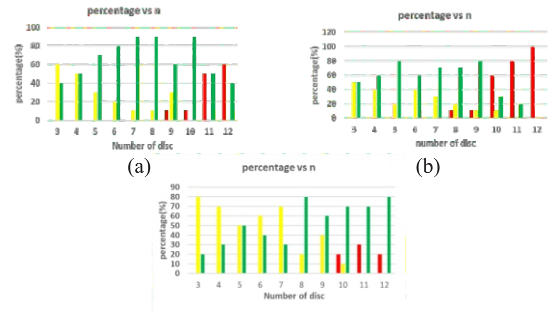


Fig. 13: Possibility of toppling per Number of the discs(4cm)

As much as the number of the discs increases the S will decrease and it makes the tower to remain standing. It is also the same in figure (13) the possibility of staying rigid increases with number of the discs. When the radius of each disc increases, possibility of toppling increases with the number of the discs (Fig. 14).



(c)

Fig. 14: . Possibility of toppling per Number of the discs, a) 8cm; b)10 cm; c) 6 cm

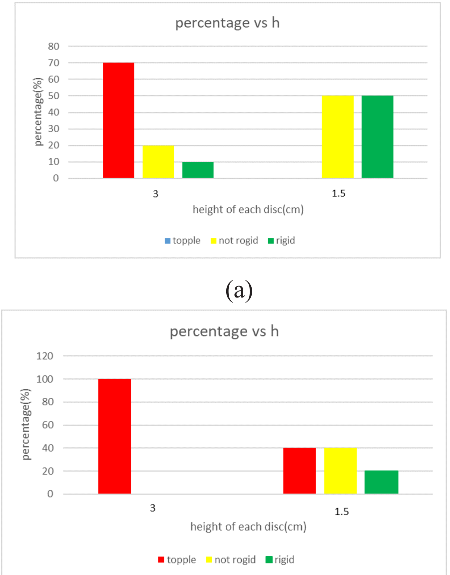
When the radius of the discs increases, the tower will have a stable situation so increasing the number of the discs will make it stable to a critical point. After that critical point, the effect of the speed of the hand will be important. When the speed of the hand is low the tower will topple because the second disc will have a high velocity otherwise the tower will remain standing.

4-2 Changing the Friction Coefficient

Some papers have been attached to the discs to change the friction coefficient. Therefore, when the friction coefficient is high the acceleration of the discs increases and the possibility of toppling increases as well and this matches the charts.

4-3 Height of Each Disc

Two discs have been attached to each other to increase the height of each disc. When the height of each disc increases, n will decrease so S decrease and the tower will topple (Fig. 15).



(b)

Fig. 15: F32. Possibility of toppling per height of the discs , a) 6cm; b) 4 cm

In all , discs in 3cm height had a higher possibility of toppling rather then 1.5 cm.

5 Ratio

A variable has been defined to conclude the effect of height and radius of the discs.

$$ratio = \frac{\text{height of the tower}}{\text{Diameter of the bottom disc}}$$

It can be concluded that by increasing the ratio, the possibility of toppling increases as well but the slope is decreasing after a while (Fig. 16) . That is because of the inertia. When the number of the discs is higher than a critical number, the inertia of the tower affects the tower a lot so the tower will not rotate when it hits the bottom disc. So the possibility of toppling will start to decrease again when the number of the discs increases.

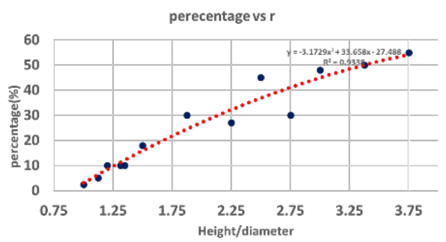


Fig 16: Possibility of toppling in each ratio

6 Conclusion

Three kinds of movement have been observed is this phenomenal. Rigid body, not rigid body and toppling. The pattern of the tower has been explained. These kind of movement have been explained according to the theory. We found in which cases there is a rigid body or not rigid body and also in which cases the tower will topple.

Some affective parameters have been investigated theoretically and experimentally and possibility of toppling have been investigated too.

When we increase μ_k the acceleration given to each disc will increase. That makes the second disc to have a greater velocity. So the distance between discs will increase and a not rigid pattern can be observed and this increases the possibility of toppling.

In cases that the radius of each disc is high, the tower will be stable. The duration that the center of mass of the tower passes through the last point of the bottom disc will be less than a critical time. But when the radius decreases the center of mass passes through the red line sooner so the tower will topple.

When the height of each disc increases, n will decrease in so S decreases and the tower will topple.

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